

The CPSA Specification:  
A Reduction System for Searching for Shapes  
in Cryptographic Protocols

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## Abstract

We describe a term reduction system that enumerates all essentially different executions possible for a cryptographic protocol. We call them the *shapes* of the protocol. Naturally occurring protocols have only finitely many, indeed very few shapes. Authentication and secrecy properties are easy to determine from them, as are attacks and anomalies. Our Cryptographic Protocols Shapes Analyzer (CPSA) program is a direct implementation of the reduction system described within, and the form of the reduction system is partially determined by the implementation.

# Chapter 1

## Introduction

The Cryptographic Protocol Shapes Analyzer (CPSA) attempts to enumerate all essentially different executions possible for a cryptographic protocol. We call them the *shapes* of the protocol. Naturally occurring protocols have only finitely many, indeed very few shapes. Authentication and secrecy properties are easy to determine from them, as are attacks and anomalies.

The shapes analysis is performed within a pure Dolev-Yao model. The CPSA program reads a sequence of problem descriptions, and prints the steps it used to solve each problem. For each input problem, CPSA is given some initial behavior, and it discovers what shapes are compatible with it. Normally, the initial behavior is from the point of view of one participant. The analysis reveals what the other participants must have done, given the participant's view.

This document specifies the CPSA program using a term reduction system. Chapter 2 describes message algebras as order-sorted quotient term algebras. Chapter 3 presents an implementation-oriented view of strand spaces. Chapter 4 details the model of the adversary. The formal definition of a partial run of a protocol is called a skeleton, and is introduced in Chapter 5.

The term reduction systems used to specify the algorithm is presented in Chapter 6, and the primitive reduction rules are in Chapter 7. The rules used to transform terms called preskeletons into skeletons are in Chapter 8.

The algorithm used to model adversarial behavior is in Chapter 9. The algorithms used to infer what else must have happened given a partial description of a run of a protocol as skeleton is in Chapters 10 and 11. Chapters 12 and 13 find most general descriptions of CPSA answers—the shapes. Finally, Chapter 14 assembles reduction rules into one system that specifies

the CPSA program.

Appendix A details the sense in which a protocol role can be viewed as an abstraction of a program. Appendix B describes a formula in the language of order-sorted first-order logic for each problem and its shapes found by CPSA. The formula is called a shape analysis sentence. The formula is modeled by all skeletons that describe full runs of a protocol when CPSA finds all the shapes for the problem.

Ideally, if there is a finite number of shapes associated with a problem statement, CPSA will find them given enough resources. In other words, the search is complete, i.e. every shape can in fact be found in a finite number of steps. The Completeness of CPSA document [8] contains our latest attempt at showing the implemented search algorithm is complete.

A CPSA release includes two other documents, the CPSA Design [9] and the CPSA Primer [10]. The design document describes details of the CPSA implementation that would clutter this one. It should be read by anyone interesting in reading and modifying the source code. The CPSA Primer provides an overview of CPSA, and may be worth reading before this document is approached.

## 1.1 Notation

A finite sequence is a function from an initial segment of the whole numbers. The length of a sequence  $X$  is  $|X|$ , and sequence  $X = \langle X(1), \dots, X(n) \rangle$  for  $n = |X|$ . Alternatively,  $\langle x_1, x_2, \dots, x_n \rangle = x_1 :: x_2 :: \dots :: x_n :: \langle \rangle$ . If  $S$  is a set, then  $S^*$  is the set of finite sequences of  $S$ , and  $S^+$  is the non-empty finite sequences of  $S$ . The concatenation of sequences  $X_0$  and  $X_1$  is  $X_0 \hat{\ } X_1$ . The prefix of sequence  $X$  of length  $n$  is  $X|_n$ .

Generally, when discussing terms, a lowercase Latin letter is used to denote a term, and an uppercase Latin letter is used to denote a set of terms or a sequence of terms.

## Chapter 2

# Order-Sorted Message Algebras

CPSA models a message by an equivalence class of terms over a signature. A sort system is used to classify messages. CPSA depends on the sort system to allow it to treat a variable that represents an asymmetric key differently from a variable that represents an arbitrary message. In particular, CPSA uses order-sorted quotient term algebras [5] for message algebras. This formalism enables the use of well-known algorithms for unification and matching in the presences of equations and sorts [11, Chapter 8].

This paper makes no attempt to provide a general introduction to order-sorted quotient term algebras. We use a message algebra called the Basic Crypto Algebra (BCA), which is the main algebra used by CPSA.

There are six BCA sorts: **mesg**, the sort of all messages, **skey**, the sort of symmetric keys, **akey**, the sort of asymmetric keys, **name**, the sort of participant names, and **text** and **data** for ordinary values. Sort **mesg** is sometimes written as  $\top$  and the other sorts are called *base sorts*. All base sorts are subsorts of **mesg**. The function symbols, or *operations*, used to form terms are given by the signature in Figure 2.1.

Each variable  $x$  used to form a term has a unique sort  $s$ , written  $x : s$ . Variable set  $X$  is an indexed set of sets of variables,  $X_s = \{x \mid x : s\}$ . For BCA,  $X_{\text{mesg}}$ ,  $X_{\text{skey}}$ ,  $X_{\text{akey}}$ ,  $X_{\text{name}}$ ,  $X_{\text{text}}$ , and  $X_{\text{data}}$  partition the set of variables in  $X$ . By abuse of notation, at times, we write  $X$  for the set of variables in  $X$ .

The Basic Crypto Quotient Term Algebra  $\mathfrak{A}$  generated by variable set  $X$  is displayed in Figure 2.2. The union of the messages in  $\mathfrak{A}$  is set of terms generated by  $X$ , and  $\mathfrak{A}$  partitions the set of terms into a set of equivalence classes induced by the equations. Terms  $t_0$  and  $t_1$  are equivalent, written



Sorts: **name**, **text**, **data**, **skey**, **akey** < **mesg**

Base sorts: **name**, **text**, **data**, **skey**, **akey**

Carried positions:  $\bullet$  denotes a carried position.

|   |                    |
|---|--------------------|
| $\{\bullet\}_{(\cdot)}$ : <b>mesg</b> $\times$ <b>mesg</b> $\rightarrow$ <b>mesg</b>    | Encryption         |
| $\#(\cdot)$ : <b>mesg</b> $\rightarrow$ <b>mesg</b>                                     | Hashing            |
| $(\bullet, \bullet)$ : <b>mesg</b> $\times$ <b>mesg</b> $\rightarrow$ <b>mesg</b>       | Pairing            |
| "...": <b>mesg</b>  | Tag constants      |
| $K_{(\cdot)}$ : <b>name</b> $\rightarrow$ <b>akey</b>                                   | Public key of name |
| $(\cdot)^{-1}$ : <b>akey</b> $\rightarrow$ <b>akey</b>                                  | Inverse of key     |
| $\text{ltk}(\cdot, \cdot)$ : <b>name</b> $\times$ <b>name</b> $\rightarrow$ <b>skey</b> | Long term key      |

Equation:  $(x^{-1})^{-1} \approx x$  for  $x$ : **akey**

Figure 2.1: Basic Crypto Signature and Equation

$$\begin{aligned}
\mathfrak{A}_{\text{skey}} &= \{\{x\} \mid x \in X_{\text{skey}}\} \cup \{\{\text{ltk}(a, b)\} \mid a \in X_{\text{name}}, b \in X_{\text{name}}\} \\
\mathfrak{A}_{\text{akey}} &= \{\{x^{-2n} \mid n \in \mathbb{N}\} \mid x \in X_{\text{akey}}\} \\
&\quad \cup \{\{x^{-2n-1} \mid n \in \mathbb{N}\} \mid x \in X_{\text{akey}}\} \\
&\quad \cup \{\{K_x^{-2n} \mid n \in \mathbb{N}\} \mid x \in X_{\text{name}}\} \\
&\quad \cup \{\{K_x^{-2n-1} \mid n \in \mathbb{N}\} \mid x \in X_{\text{name}}\} \\
\mathfrak{A}_{\text{name}} &= \{\{x\} \mid x \in X_{\text{name}}\} \\
\mathfrak{A}_{\text{text}} &= \{\{x\} \mid x \in X_{\text{text}}\} \\
\mathfrak{A}_{\text{data}} &= \{\{x\} \mid x \in X_{\text{data}}\} \\
\mathfrak{B} &= \mathfrak{A}_{\text{skey}} \cup \mathfrak{A}_{\text{akey}} \cup \mathfrak{A}_{\text{name}} \cup \mathfrak{A}_{\text{text}} \cup \mathfrak{A}_{\text{data}} \\
\mathfrak{A}^0 &= \mathfrak{B} \cup \{\{x\} \mid x \in X_{\text{mesg}}\} \cup \{\{x\} \mid x \text{ is a tag constant}\} \\
\mathfrak{A}^{n+1} &= \mathfrak{A}^n \cup \{\{(t_0, t_1) \mid t_0 \in T_0, t_1 \in T_1\} \mid T_0 \in \mathfrak{A}^n, T_1 \in \mathfrak{A}^n\} \\
&\quad \cup \{\{\{t_0\}_{t_1} \mid t_0 \in T_0, t_1 \in T_1\} \mid T_0 \in \mathfrak{A}^n, T_1 \in \mathfrak{A}^n\} \\
&\quad \cup \{\{\#t \mid t \in T\} \mid T \in \mathfrak{A}^n\} \\
\mathfrak{A} = \mathfrak{A}_{\text{mesg}} &= \bigcup_{n \in \mathbb{N}} \mathfrak{A}^n
\end{aligned}$$

Figure 2.2: BCA Messages  $\mathfrak{A}$  and Atoms  $\mathfrak{B}$

$t_0 \equiv t_1$ , iff  $t_0 \in T \wedge t_1 \in T$  for some  $T \in \mathfrak{A}$ . The canonical representative of a message is the  $t$  in  $\{t' \mid t' \equiv t\}$  with the fewest occurrences of the  $(\cdot)^{-1}$  operation.

Keys, names, data, and texts in the algebra are called *atoms* and are members of  $\mathfrak{B}$ . We write  $t: B$  iff  $\{t' \mid t' \equiv t\} \in \mathfrak{B}$ . Note that encryption is defined with an encryption key of sort **mesg**. When the encryption key is of sort **akey** this is meant to model asymmetric encryption: otherwise, this models symmetric encryption. Note that even complex messages such as encryptions can be used as encryption keys in the symmetric sense.

To find the decryption key associated with an encryption, one must exclude the case in which the key is a variable of sort **mesg**, as there is no way to determine if the encryption operation denotes symmetric or asymmetric encryption. Therefore, the decryption key associated with encryption key  $t$  is  $inv(t)$ .

$$inv(t) = \begin{cases} invk(t) & \text{if } t: \mathbf{akey}; \\ \text{undefined} & \text{if } t \text{ is a variable of sort } \mathbf{mesg}; \\ t & \text{otherwise.} \end{cases}$$

An important property possessed by the algebra is that for all  $T \in \mathfrak{A}$ , if there are any encryptions in  $T$  then all members of  $T$  are encryptions. As a result, a message can be identified as representing an encryption and if it is, decomposed into its plaintext and its decryption key. This property is a consequence of the fact that equations relate atoms, not arbitrary messages. A similar property holds for pairs and hashes. A hash is treated as a kind of encryption in which the term that is hashed is the encryption key.

We write  $\mathfrak{A}_X$  when it is important to identify the variable set  $X$  that generates the algebra. Given two variable sets  $X$  and  $Y$ , a *substitution* is an order-sorted map  $\sigma: X \rightarrow \mathfrak{A}_Y$  such that  $\sigma(x) \neq x$  for only finitely many elements of  $X$ . For a substitution  $\sigma$ , the *domain* is the set of variables  $Dom(\sigma) = \{x \mid \sigma(x) \neq x\}$  and the *range* is the set  $Ran(\sigma) = \{\sigma(x) \mid x \in Dom(\sigma)\}$ . Substitution  $\sigma_0$  is *more general than*  $\sigma_1$ , written  $\sigma_0 \trianglelefteq \sigma_1$ , if there exists a substitution  $\sigma_2$  such that  $\forall x \sigma_1(x) \equiv \sigma_2(\sigma_0(x))$ . Given a substitution  $\sigma: X \rightarrow \mathfrak{A}_Y$ , the unique homomorphism  $\sigma^*: \mathfrak{A}_X \rightarrow \mathfrak{A}_Y$  induced by  $\sigma$  is also denoted  $\sigma$ .

A *position*  $p$  is a finite sequence of whole numbers. The term in  $t$  that

occurs at  $p$ , written  $t @ p$ , is:

$$\begin{aligned}
t @ \langle \rangle &= t; \\
(t_1, t_2) @ i &:: p = t_i @ p \text{ for } i \in \{1, 2\}; \\
\{t_1\}_{t_2} @ i &:: p = t_i @ p \text{ for } i \in \{1, 2\}; \\
t^{-1} @ 1 &:: p = t @ p.
\end{aligned}$$

A term  $t$  occurs in term  $t'$  if  $t = t' @ p$  for some  $p$ . A message  $T$  occurs in message  $T'$  if the canonical representative of  $T$  occurs in the canonical representative of  $T'$ .

A carried term is one that can be extracted from a message reception assuming plaintext is extractable from encryptions. The positions at which term  $t$  is carried in  $t'$  is  $carpos(t, t')$ , where

$$carpos(t, t') = \begin{cases} \{\langle \rangle\} & \text{if } t' \equiv t, \text{ else} \\ \{1 :: p \mid p \in carpos(t, t_1)\} & \text{if } t' = \{t_1\}_{t_2}, \text{ else} \\ \{i :: p \mid i \in \{1, 2\}, p \in carpos(t, t_i)\} & \text{if } t' = (t_1, t_2) \text{ else} \\ \emptyset & \text{otherwise.} \end{cases}$$

Term  $t$  carries  $t'$  if  $carpos(t', t)$  is not empty, and  $t' \sqsubseteq t$  when  $t'$  is carried by  $t$ . Note that for all terms  $t_0, t_1, t'_0, t'_1$ , if  $t_0 \equiv t_1$  and  $t'_0 \equiv t'_1$ , then  $carpos(t_0, t'_0) = carpos(t_1, t'_1)$ . We write  $t' \sqsubseteq_p t$  when  $p \in carpos(t', t)$  and  $t @ p \equiv t'$ .

In what follows, we will often conflate a term with the message of which it is a member, and use lowercase letters to denote both.

# Chapter 3

## Strand Spaces and Bundles

When using strand space theory, one normally hypothesizes the existence of a single global strand space. This is a very reasonable assumption for theoretical analysis, but from the point view of an implementer, it turns out that it is better to assume there are many local strand spaces and the design specification task is to describe the relations between these local spaces. Our reformulation of strand space notation provides an implementation oriented way of describing the concept of a local strand space, and a direct link between from algorithm specification to the data structures used in the implementation.

A run of a protocol is viewed as an exchange of messages by a finite set of local sessions of the protocol. Each local session is called a *strand*. The behavior of a strand, its *trace*, is a sequence of messaging events. An *event* is either a message transmission or a reception. Outbound message  $t \in \mathfrak{A}_X$  is written as  $+t$ , and inbound message  $t$  is written as  $-t$ . The set of traces over  $\mathfrak{A}_X$  is  $\mathfrak{C}_X = (\pm\mathfrak{A}_X)^+$ . A message *originates* in a trace if it is carried by some event and the first event in which it is carried is outbound. A message is *gained* by a trace if it is carried by some event and the first event in which it is carried is inbound. A message is *acquired* by a trace if it first occurs in a reception event and is also carried by that event.

Abstractly, a strand space is a multiset of traces, but since we wish to name each element, a *strand space*  $\Theta_X$  over algebra  $\mathfrak{A}_X$  is defined to be a sequence of traces in  $\mathfrak{C}_X$ . A strand  $s$  is a member of the domain of  $\Theta_X$ , and its trace is  $\Theta_X(s)$ . In a strand space, the elements of the generator set  $X$  denote atomic message elements, such as keys, and not composite messages, such as encryptions and pairs. Therefore, the sort of every variable in  $X$  is a base sort.

Message events occur at nodes in a strand space. For each strand  $s$ , there is a node for every event in  $\Theta(s)$ . The *nodes* of strand space  $\Theta$  are  $\{(s, i) \mid s \in \text{Dom}(\Theta), 1 \leq i \leq |\Theta(s)|\}$ , the event at a node is  $\text{evt}_\Theta(s, i) = \Theta(s)(i)$ , and the message at a node is  $\text{msg}_\Theta(s, i) = m$  such that  $\text{evt}_\Theta(s, i) = \pm m$ . Just as a position names a subterm within another term, a strand names a trace within a strand space, and a node names an event in a strand space. The relation  $\Rightarrow$  defined by  $\{(s, i) \Rightarrow (s, i + 1) \mid s \in \text{Dom}(\Theta), 1 \leq i < |\Theta(s)|\}$  is called the *strand succession relation*.

A *bundle* in strand space  $\Theta$  is a finite directed acyclic graph  $\Upsilon(\Theta, \rightarrow)$ , where the vertices are the nodes of  $\Theta$ , and an edge represents communication ( $\rightarrow$ ) or strand succession ( $\Rightarrow$ ). For communication, if  $n_0 \rightarrow n_1$ , then there is a message  $t$  such that  $\text{evt}_\Theta(n_0) = +t$  and  $\text{evt}_\Theta(n_1) = -t$ . For each reception node  $n_1$ , there is a unique transmission node  $n_0$  with  $n_0 \rightarrow n_1$ .

Each acyclic graph has a transitive asymmetric relation  $\prec$  on its vertices. The relation specifies the causal ordering of nodes in a bundle. Relation  $R$  on set  $S$  is *asymmetric* iff  $x R y$  implies not  $y R x$  for all distinct  $x, y \in S$ . An asymmetric transitive binary relation is also called a strict partial order.

An atom *uniquely originates* in a bundle if it originates in the trace of exactly one strand. An atom is *non-originating* in a bundle if it originates on no strand, but each of its variables occurs in some strand's trace.

In a run of a protocol, the behavior of each strand is constrained by a role in a protocol. Adversarial strands are constrained by roles as are non-adversarial strands. A protorole over  $\mathfrak{A}_Y$  is  $\mathfrak{r}_Y(C, N, U)$ , where  $C \in \mathfrak{C}_Y$ ,  $N \subseteq \mathfrak{B}_Y$ , and  $U \subseteq \mathfrak{B}_Y$ . The trace of the role is  $C$ , its non-origination assumptions are  $N$ , and its unique origination assumptions are  $U$ . A protorole is a *role* if (1)  $t \in N$  implies  $t$  is not carried in  $C$ , and all variables in  $N$  occur in  $C$ , (2)  $t \in U$  implies  $t$  originates in  $C$ , (3) if variable  $x$  occurs in  $C$  then  $x$  is an atom or it is acquired in  $C$ , and (4) the trace of a role may not match the pattern  $\langle -t, +t, \dots \rangle$ . This is to ensure that listeners, which are introduced on Page 13, cannot be confused with protocol constrained strands. A *protocol* is a set of roles. Let  $\text{Vars}(P)$  be the set of variables that occur in the traces of the roles in protocol  $P$ .

A bundle  $\Upsilon(\Theta_X, \rightarrow)$  is a *run of protocol*  $P$  if there is a role mapping  $rl: \Theta_X \rightarrow P$  that satisfies properties for each  $s \in \text{Dom}(\Theta_X)$ . Assuming  $rl(s) = \mathfrak{r}_Y(C, N, U)$  and  $X$  and  $Y$  share no variables, and let  $h = |\Theta_X(s)|$ , the properties are (1)  $h \leq |C|$ , (2) there is a homomorphism  $\sigma: \mathfrak{A}_Y \rightarrow \mathfrak{A}_X$  such that  $\sigma \circ C|_h = \Theta_X(s)$ , (3)  $\text{Dom}(\sigma)$  is the set of variables that occur in  $C|_h$ , (4) if the variables in  $t \in N$  occur in  $\text{Dom}(\sigma)$ , then  $\sigma(t)$  is non-originating in

$\Upsilon(\Theta_X, \rightarrow)$ , and (5) if  $t \in U$  originates at index  $i$  in  $C$ , and  $i \leq h$ , then  $\sigma(t)$  uniquely originates in  $\Upsilon(\Theta_X, \rightarrow)$  at node  $(s, i)$ . Origination assumptions in bundles specified by roles are called *inherited origination assumptions*.

# Chapter 4

## Adversary Model

A fixed set of penetrator roles encodes the adversary model associated with a message algebra. For the Basic Crypto Algebra, there are eight roles. Each role makes no origination assumptions, and the trace of each role is given in Figure 4.1. The first line of the figure specifies many traces, one for each base sort, and a trace for each tag.

A strand exhibits non-adversarial behavior when its role is not a penetrator role. A non-adversarial strand is called a *regular* strand as is its role.

The penetrator cannot use a non-originating atom to encrypt or decrypt a message, because every key it uses must be carried in a message. Consider a uniquely originating atom that originates on a regular strand. The penetrator cannot make the atom using a create role, because the atom would originate in more than one trace. Therefore, the penetrator can use a uniquely originating atom to encrypt or decrypt a message only if it is transmitted by a regular strand unprotected by encryption.

|                         |                                    |   |
|-------------------------|------------------------------------|---|
| Create( $z: B$ )        | $\langle +z \rangle$               | $\langle +\text{"..."} \rangle$         |
| Pair( $x, y: \top$ )    | $\langle -x, -y, +(x, y) \rangle$  | $\langle -(x, y), +x, +y \rangle$       |
| Encrypt( $x, y: \top$ ) | $\langle -x, -y, +\{x\}_y \rangle$ | $\langle -\{x\}_y, -inv(y), +x \rangle$ |
| Hash( $x: \top$ )       | $\langle -x, +\#x \rangle$         |   |

Figure 4.1: Basic Crypto Algebra Penetrator Role Traces

# Chapter 5

## Skeletons

The details of penetrator behavior are abstracted away when performing protocol analysis. The abstracted description of a bundle is called a realized skeleton, which is defined using a protoskeleton. A *protoskeleton* over  $\mathfrak{A}_X$  is  $\mathbf{k}_X(\mathit{rl}, P, \Theta_X, \prec, N, U)$ , where  $\mathit{rl}: \text{Dom}(\Theta_X) \rightarrow P$  is a role map, the sets  $X$  and  $\text{Vars}(P)$  are disjoint,  $\Theta_X$  is a sequence of traces in  $\mathfrak{C}_X$ ,  $\prec$  is a relation on the nodes in  $\Theta_X$ ,  $N \subseteq \mathfrak{B}_X$  are its non-origination assumptions, and  $U \subseteq \mathfrak{B}_X$  are its unique origination assumptions. Unlike a strand space, the sort of a variable in  $X$  need not be a base sort.

Assume the strands in bundle  $\Upsilon(\Theta_X, \rightarrow)$  have been permuted so that regular strands precede penetrator strands in sequence  $\Theta_X$ , and  $\mathit{rl}$  demonstrates the bundle is a run of protocol  $P$ . Let  $P'$  be  $P$  without penetrator roles. Skeleton  $\mathbf{k}_X(\mathit{rl}', P', \Theta'_X, \prec, N, U)$  *realizes* the bundle if  $\mathit{rl}'$  and  $\Theta'_X$  are the truncations of  $\mathit{rl}$  and  $\Theta_X$  respectively that omit penetrator strands from their domains,  $\prec$  is the transitive asymmetric relation associated with the bundle without penetrator nodes,  $N$  is the set of non-originating atoms with variables that occur in  $\Theta'_X$ , and  $U$  is the set of atoms that uniquely originate and are carried by some regular event.

A protoskeleton  $\mathbf{k}_X(\mathit{rl}, P, \Theta_X, \prec, N, U)$  is a *preskeleton* if the following properties hold.

1. Sequence  $\mathit{rl}$  demonstrates that the strands in  $\text{Dom}(\Theta_X)$  satisfy the conditions for being a part of a run of protocol  $P$ .
2. Relation  $\prec$  is transitive, asymmetric, and includes the strand succession relation ( $\Rightarrow$ ).



3. Each atom in  $N$  is carried by no event, and each variable in the atom occurs in some event.
4. Each atom in  $U$  is carried by some event.
5.  $N$  includes the non-originating atoms inherited from roles via the role map.
6.  $U$  includes the uniquely originating atoms inherited from roles via the role map.

Let  $\mathcal{O}_k(t)$  be the set of nodes at which  $t$  originates in preskeleton  $k$ , and  $\mathcal{G}_k(t)$  be the set of nodes at which  $t$  is gained in  $k$ . Preskeleton  $k_X(rl, P, \Theta_X, \prec, N, U)$  is a *skeleton* if each atom in  $U$  originates on at most one strand, and the node of origination precedes each node that gains the atom, i.e. for every  $t \in U$ ,  $n_0 \in \mathcal{O}_k(t)$  and  $n_1 \in \mathcal{G}_k(t)$  implies  $n_0 \prec n_1$ .

Let  $k_0 = k_X(rl_0, P, \Theta_0, \prec_0, N_0, U_0)$  and  $k_1 = k_Y(rl_1, P, \Theta_1, \prec_1, N_1, U_1)$  be preskeletons. There is a *preskeleton homomorphism* from  $k_0$  to  $k_1$  if  $\phi$  and  $\sigma$  are maps with the following properties:

1.  $\phi$  maps strands of  $k_0$  into those of  $k_1$ , and nodes as  $\phi((s, i)) = (\phi(s), i)$ , that is  $\phi$  is in  $Dom(\Theta_0) \rightarrow Dom(\Theta_1)$ ;
2.  $\sigma: \mathfrak{A}_X \rightarrow \mathfrak{A}_Y$  is a message algebra homomorphism;
3.  $n \in nodes(\Theta_0)$  implies  $\sigma(evt_{\Theta_0}(n)) = evt_{\Theta_1}(\phi(n))$ ;
4.  $n_0 \prec_0 n_1$  implies  $\phi(n_0) \prec_1 \phi(n_1)$ ;
5.  $\sigma(N_0) \subseteq N_1$ ;
6.  $\sigma(U_0) \subseteq U_1$ ;
7.  $t \in U_0$  implies  $\phi(\mathcal{O}_{k_0}(t)) \subseteq \mathcal{O}_{k_1}(\sigma(t))$ .

A homomorphism is *strandwise injective* if its strand map is injective. Two preskeletons are isomorphic if they are related by strandwise injective homomorphism in both directions. A homomorphism is *nodewise isomorphic* if the strand map  $\phi$  implies a bijection on nodes, and  $n_0 \prec_1 n_1$  implies  $\phi^{-1}(n_0) \prec_0 \phi^{-1}(n_1)$ . A skeleton is *realized* if there is a nodewise isomorphic homomorphism from it to a skeleton that realizes a bundle, and message component of the homomorphism is injective.

Our formalism requires that every protocol include a listener role of the form:  $l_{sn}(x: \top) = r(\langle -x, +x \rangle, \emptyset, \emptyset)$ . Instances of this role are sometimes used to make penetrator derived messages visible in skeletons. We say skeleton  $k$  *realizes modulo listeners* bundle  $\Upsilon(\Theta, \rightarrow)$  if  $k$  realizes  $\Upsilon(\Theta', \rightarrow')$  and  $\Upsilon(\Theta, \rightarrow)$  is the result of removing listener strands, and adjusting the communication ordering  $\rightarrow$  appropriately.

The set of bundles denoted by preskeleton  $k$ ,  $\llbracket k \rrbracket$ , is:

$$\llbracket k \rrbracket = \{ \Upsilon \mid k \xrightarrow{\phi, \sigma} k' \text{ and } k' \text{ realizes modulo listeners } \Upsilon \}$$

A CPSA algorithm is *complete* if when given a preskeleton  $k$ , either the algorithm diverges, or else it terminates and produces a finite set of realized skeletons  $K$ , such that  $\llbracket k \rrbracket = \bigcup_{k' \in K} \llbracket k' \rrbracket$ .

## 5.1 Blanchet's Simple Example Protocol

The following protocol is a simplified version of the Denning-Sacco key distribution protocol [3] due to Bruno Blanchet.

$$\begin{aligned} A \rightarrow B &: \{ \{ \{ s \}_{a^{-1}} \}_b \\ B \rightarrow A &: \{ \{ d \}_s \} \end{aligned}$$

Symmetric key  $s$  is freshly generated, asymmetric keys  $a^{-1}$  and  $b^{-1}$  are uncompromised, and the goal of the protocol is to keep data  $d$  secret. The protocol was constructed with a known flaw for expository purposes.

This CPSA description of the protocol has an initiator and a responder role.

$$\begin{aligned} \mathit{init}(a, b: \mathbf{A}, s: \mathbf{S}, d: \mathbf{D}) &= r(\langle +\{ \{ s \}_{a^{-1}} \}_b, -\{ \{ d \}_s \}, \emptyset, \emptyset) \\ \mathit{resp}(a, b: \mathbf{A}, s: \mathbf{S}, d: \mathbf{D}) &= r(\langle -\{ \{ s \}_{a^{-1}} \}_b, +\{ \{ d \}_s \}, \emptyset, \emptyset) \end{aligned}$$

where we use  $\mathbf{A}$  for sort **akey**,  $\mathbf{S}$  for sort **skey**, and  $\mathbf{D}$  for sort **data** to save space. The algebra for the initiator role is generated from  $X$ , where  $X_{\mathbf{A}} = \{a, b\}$ ,  $X_{\mathbf{S}} = \{s\}$ ,  $X_{\mathbf{D}} = \{d\}$ ,  $X_{\mathbf{text}} = \emptyset$ ,  $X_{\mathbf{name}} = \emptyset$ , and  $X_{\top} = \emptyset$ .

An interesting point of view for analysis is to see if the authentication goals of the initiator are met. To do so, we assume there was full length run of an initiator strand, and let CPSA determine what else must have happened.

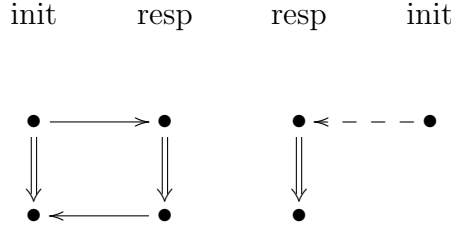


Figure 5.1: Shapes for Blanchet's Protocol

Let variable set  $Y = a, b: \mathbf{A}, s: \mathbf{S}, d: \mathbf{D}$ . The point-of-view skeleton is:

|   |                    |
|---|--------------------|
| $k_Y(\langle \mathit{init}(a_0, b_0, s_0, d_0) \rangle,$                    | Role map           |
| $\{\mathit{init}(a_0, b_0, s_0, d_0), \mathit{resp}(a_1, b_1, s_1, d_1)\},$ | Protocol           |
| $\langle \langle +\{\{s\}_{a^{-1}}\}_b, -\{d\}_s \rangle \rangle,$          | Traces             |
| $\emptyset,$  | Node orderings     |
| $\{a^{-1}, b^{-1}\},$   | Non-origination    |
| $\{s\})$  | Unique origination |

where the variable set that generates the algebra for the initiator and responder roles have been renamed so as to avoid conflicts with the variable set  $Y$  used by the skeleton.

The skeleton produced by CPSA for this problem follows. Notice that the two strands agree on the key  $b$  used in the outermost encryption of their first message, and indication that the authentication goals of the initiator are met. See Figure 5.1 to see the structure of the shape.

|   |                    |
|---|--------------------|
| $k_Y(\langle \mathit{init}(a_0, b_0, s_0, d_0), \mathit{resp}(a_1, b_1, s_1, d_1) \rangle,$ | Role map           |
| $\{\mathit{init}(a_0, b_0, s_0, d_0), \mathit{resp}(a_1, b_1, s_1, d_1)\},$                 | Protocol           |
| $\langle \langle +\{\{s\}_{a^{-1}}\}_b, -\{d\}_s \rangle \rangle,$                          | Traces             |
| $\langle -\{\{s\}_{a^{-1}}\}_b, +\{d\}_s \rangle \rangle,$                                  |                    |
| $\{(1, 1) \prec (2, 1), (2, 2) \prec (1, 2)\},$   | Node orderings     |
| $\{a^{-1}, b^{-1}\},$   | Non-origination    |
| $\{s\})$  | Unique origination |

The homomorphism from the point-of-view skeleton to the shape is

$$(\langle 1 \rangle, \{a \mapsto a, b \mapsto b, s \mapsto s, d \mapsto d\}).$$

An analysis of the authentication goals for the responder shows the flaw built into the protocol. To do the analysis, assume there was a full length run

of a responder strand, and let CPSA determine what else must have happened. In this case, the point-of-view skeleton is:

|  |                    |
|--|--------------------|
| $k_Y(\langle resp(a_0, b_0, s_0, d_0) \rangle,$                    | Role map           |
| $\{init(a_0, b_0, s_0, d_0), resp(a_1, b_1, s_1, d_1)\},$          | Protocol           |
| $\langle \langle -\{\{s\}_{a^{-1}}\}_b, +\{d\}_s \rangle \rangle,$ | Traces             |
| $\emptyset,$   | Node orderings     |
| $\{a^{-1}, b^{-1}\},$  | Non-origination    |
| $\{s\}$  | Unique origination |

The shape generated by CPSA follows. An early indication of a problem is the variable set for the shape has three asymmetric keys. Let variable set  $Z = a, b, b' : A, s : S, d : D$ . The shape is:

|   |                              |
|---|------------------------------|
| $k_Z(\langle resp(a_0, b_0, s_0, d_0), init(a_1, b_1, s_1, d_1) \rangle,$ | Role map                     |
| $\{init(a_0, b_0, s_0, d_0), resp(a_1, b_1, s_1, d_1)\},$                 | Protocol                     |
| $\langle \langle -\{\{s\}_{a^{-1}}\}_b, +\{d\}_s \rangle \rangle,$        | Traces                       |
| $\langle +\{\{s\}_{a^{-1}}\}_{b'} \rangle \rangle,$                       | <i>Note key is b' not b!</i> |
| $\{(2, 1) \prec (1, 1)\},$  | Node orderings               |
| $\{a^{-1}, b^{-1}\},$   | Non-origination              |
| $\{s\}$   | Unique origination           |

Notice that the two strands do not agree on the key used in the outermost encryption of their first message—an authentication failure. To see that the authentication failure leads to the failure to protect the secrecy of data  $d$ , the protocol is analyzed using the following point-of-view:

|  |                    |
|--|--------------------|
| $k_Y(\langle resp(a_0, b_0, s_0, d_0), lsn(x) \rangle,$                                    | Role map           |
| $\{init(a_0, b_0, s_0, d_0), resp(a_1, b_1, s_1, d_1), lsn(x)\},$                          | Protocol           |
| $\langle \langle -\{\{s\}_{a^{-1}}\}_b, +\{d\}_s \rangle, \langle -d, +d \rangle \rangle,$ | Traces             |
| $\{(1, 2) \prec (2, 1)\},$   | Node orderings     |
| $\{a^{-1}, b^{-1}\},$  | Non-origination    |
| $\{s, d\}$   | Unique origination |

CPSA finds a shape that shows how data  $d$  is revealed to the adversary.

## 5.2 Dolev-Yao Example 1.3

The intended run of the protocol in the Dolev-Yao Example 1.3 is:

$$\begin{aligned} A &\rightarrow B : \{\{m\}_b, a\}_b \\ B &\rightarrow A : \{\{m\}_a, b\}_a \end{aligned}$$

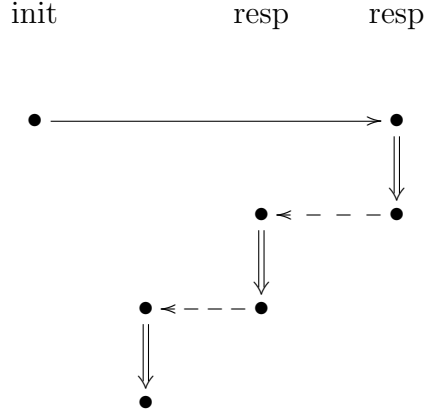


Figure 5.2: Dolev-Yao Example 1.3 Shape

assuming text  $m$  is freshly generated, and asymmetric keys  $a^{-1}$  and  $b^{-1}$  are uncompromised.

The CPSA description of the protocol also has an initiator and a responder role.

$$\begin{aligned} \mathit{init}(a, b: \mathbf{A}, m: \mathbf{D}) &= r(\langle +\{\{m\}_b, a\}_b, -\{\{m\}_a, b\}_a \rangle, \emptyset, \emptyset) \\ \mathit{resp}(a, b: \mathbf{A}, m: \mathbf{T}) &= r(\langle -\{\{m\}_b, a\}_b, +\{\{m\}_a, b\}_a \rangle, \emptyset, \emptyset) \end{aligned}$$

An interesting point of view for analysis is to see if  $m$  is kept secret after the initiator sends its message. Let variable set  $Z = a, b: \mathbf{A}, m: \mathbf{D}$ . The initial scenario preskeleton is:

|  |                    |
|--|--------------------|
| $k_Z(\langle \mathit{init}(a_0, b_0, m_0), \mathit{lsn}(x) \rangle,$               | Role map           |
| $\{\mathit{init}(a_0, b_0, m_0), \mathit{resp}(a_1, b_1, m_1), \mathit{lsn}(x)\},$ | Protocol           |
| $\langle \langle +\{\{m\}_b, a\}_b, \langle -m \rangle \rangle,$                   | Traces             |
| $\emptyset,$   | Node orderings     |
| $\{a^{-1}, b^{-1}\},$  | Non-origination    |
| $\{m\})$   | Unique origination |

where the variable set that generates the algebra for the initiator role has been renamed so as to avoid conflicts with the variable set  $Z$  used by the preskeleton.

CPSA determines  $m$  is not kept secret by producing the shape in Figure 5.2. The added strands in the shape are instances of responder roles.

init          resp          resp

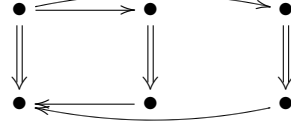


Figure 5.3: Exercise Skeleton

The strands in the shape are:

$$\begin{aligned}
 & \langle +\{\{m\}_b, a\}_b \rangle \\
 & \langle -m \rangle \\
 & \langle -\{\{m\}_b, a'\}_b, +\{\{m\}_{a'}, b\}_{a'} \rangle \\
 & \langle -\{\{\{m\}_b, a\}_b, a''\}_b, +\{\{\{m\}_b, a\}_{a''}, b\}_{a''} \rangle
 \end{aligned}$$

The non-origination and unique origination assumptions are as they are in the initial scenario preskeleton. An interesting exercise left for the reader is to produce a bundle that is realized by the shape.

### 5.3 Exercise

Consider the following roles.

$$\begin{aligned}
 \mathit{init}(a, b: \mathbf{A}) &= r(\langle +(a, b), -(b, a) \rangle, \emptyset, \emptyset) \\
 \mathit{resp}(a, b: \mathbf{A}) &= r(\langle -(a, b), +(b, a) \rangle, \emptyset, \emptyset)
 \end{aligned}$$

Let  $X = x, y: \mathbf{A}$  and  $k = k_X(\langle \mathit{init}(a, b), \mathit{resp}(a, b), \mathit{resp}(a, b) \rangle, \{\mathit{init}(a, b), \mathit{resp}(a, b)\}, \langle \langle +(x, y), -(y, x) \rangle, \langle -(x, y), +(y, x) \rangle, \langle -(x, y), +(y, x) \rangle \rangle, \text{Node ordering in Figure 5.3}, \emptyset, \emptyset)$

What is  $\llbracket k \rrbracket$ ?

One member is shown in Figure 5.4.

init  $\langle +(x, y), -(y, x) \rangle$   
 resp  $\langle -(x, y), +(y, x) \rangle$   
 resp  $\langle -(x, y), +(y, x) \rangle$   
 pair  $\langle -(y, x), -(y, x), +((y, x), (y, x)) \rangle$   
 sep  $\langle -((y, x), (y, x)), +(y, x) \rangle$

init      resp      resp      pair      sep

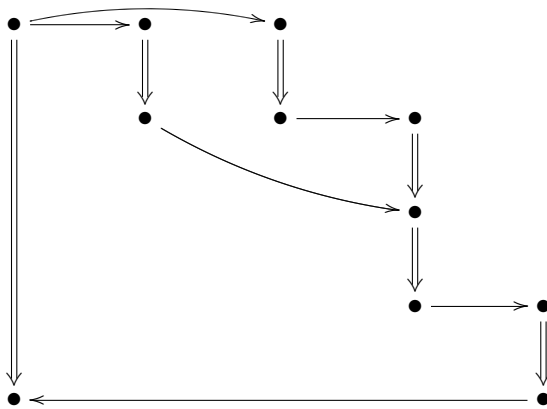


Figure 5.4: A Bundle Realized by the Example Skeleton

## 5.4 External Syntax and Instances

The external syntax used by CPSA is a little different than what has been described here. In the external syntax, the trace and the role associated with a strand is specified by an *instance*. An instance is of the form  $i(r, h, \sigma)$ , where  $r$  is a role,  $h$  specifies the length of a trace instantiated from the role, and  $\sigma$  specifies how to instantiate the variables in the role to obtain the trace. Thus when  $r = r_Y(C, N, U)$ , the trace associated with  $i(r, h, \sigma)$  is  $\sigma \circ C|_h$ . An instance is well-formed if  $1 \leq h \leq |C|$ , and  $Dom(\sigma)$  is the set of variables that occur in  $C|_h$ .

In the external syntax, the role map and sequence of traces are replaced by a sequence of instances. So for preskeleton  $\mathbf{k}_X(rl, P, \Theta_X, \prec, N, U)$ , the external syntax is  $\mathbf{k}_X(P, I, \prec, N, U)$ , where for each  $s \in Dom(\Theta_x)$ ,  $I(s) = i(r, h, \sigma)$ ,  $r = rl(s)$ , and the trace of  $i(r, h, \sigma)$  is  $\Theta_X(s)$ .



# Chapter 6

## Algorithms as Reduction Systems

Algorithms in this paper are specified as abstract reduction systems [1, Chapter 2]. A reduction system is a pair  $(A, \rightarrow)$ , where reduction  $\rightarrow$  is a binary relation  $\rightarrow \subseteq A \times A$ . Element  $x \in A$  is a *normal form* if there is no  $y$  such that  $x \rightarrow y$ . The transitive closure of  $\rightarrow$  is  $\rightarrow^+$ . The reflexive transitive closure of  $\rightarrow$  is  $\rightarrow^*$ . A reduction is confluent if  $x \rightarrow^* y_0$  and  $x \rightarrow^* y_1$  implies there is a  $z$  such that  $y_0 \rightarrow^* z$  and  $y_1 \rightarrow^* z$ . A reduction is terminating if there are no infinite descending chains. A reduction is convergent if it is confluent and terminating.

Let  $\mathfrak{K}$  be the set of preskeletons. Algorithms are specified as reduction systems of the form  $(\mathfrak{K}, \rightarrow)$ , which are then used to specify a related setwise reduction system of the form  $(2^{\mathfrak{K}}, \twoheadrightarrow)$ . Setwise reduction systems are the ones with the interesting normal forms and confluence properties. In a setwise reduction system, reduction rewrites one element of a set to a set of elements.

**Definition 6.1** (Setwise Reduction System). The *setwise reduction system* of binary relation  $\twoheadrightarrow \subseteq \mathfrak{K} \times 2^{\mathfrak{K}}$  is a reduction system  $(2^{\mathfrak{K}}, \twoheadrightarrow)$ , where for each  $K_0 \in 2^{\mathfrak{K}}$ ,  $K_0 \twoheadrightarrow K_1$  if for some  $k_0 \in K_0$ ,  $k_0 \rightsquigarrow K_2$ ,  $K_1 = K_2 \cup (K_0 \setminus \{k_0\})$ , and  $K_1 \neq K_0$ .

The CPSA algorithm will be specified as a setwise term reduction system, where the initial problem is given a singleton in  $2^{\mathfrak{K}}$ , and the answers computed by an implementation of the algorithm are a normal form of the setwise reduction relation  $\twoheadrightarrow_k$  defined in Chapter 14.

In what follows the relation  $k \rightsquigarrow K$  is defined in terms of  $\rightarrow \subseteq \mathfrak{K} \times \mathfrak{K}$  by specifying  $\{k\} \twoheadrightarrow K$  using  $\rightarrow$ , so the  $\rightsquigarrow$  relation is not explicitly defined.

We regard sets of preskeletons as factored by isomorphism, where each set has at most one representative of the equivalence class of isomorphic preskeletons. The definition of isomorphic preskeletons is given on Page 12.

The CPSA Design [9] describes an extension of a message algebra signature that models the data structures used in the CPSA program. The terms over the extended signature include ones that model preskeletons. Sets of terms of sort preskeleton are the domain of our setwise reduction systems.

# Chapter 7

## Primitive Preskeleton Operators

There are four primitive operators on preskeletons used by CPSA to solve authentication tests. Each operator is a partial map from preskeletons to preskeletons.

**Definition 7.1** (Substitution Operator). For order-sorted substitution  $\sigma: X \rightarrow \mathfrak{A}_Y$ , the operator  $\mathbb{S}_\sigma$  is:

$$\mathbb{S}_\sigma(\mathbf{k}_X(rl, P, \Theta_X, \prec, N, U)) = \mathbf{k}_Y(rl, P, s \mapsto \sigma \circ \Theta_X(s), \prec, \sigma(N), \sigma(U))$$

For  $k' = \mathbb{S}_\sigma(k)$ , there is a homomorphism from  $k$  to  $k'$  only if for all  $t \in U_k$ ,  $\mathcal{O}_k(t) \subseteq \mathcal{O}_{k'}(\sigma(t))$ . The structure preserving maps associated with the homomorphism are  $\phi_{\text{id}}$  and  $\sigma$ .

**Definition 7.2** (Compression Operator). For distinct strands  $s$  and  $s'$ , operator  $\mathbb{C}_{s,s'}$  compresses strand  $s$  into  $s'$ .

$$\mathbb{C}_{s,s'}(\mathbf{k}_X(rl, P, \Theta_X, \prec, N, U)) = \mathbf{k}_X(rl \circ \phi'_s, P, \Theta_X \circ \phi'_s, \prec', N, U)$$

where

$$\phi'_s(j) = \begin{cases} j + 1 & \text{if } j \geq s \\ j & \text{otherwise,} \end{cases}$$

relation  $\prec'$  is the transitive closure of  $\phi_{s,s'}(\prec)$ , and

$$\phi_{s,s'}(j) = \begin{cases} \phi_s(s') & \text{if } j = s \\ \phi_s(j) & \text{otherwise} \end{cases}$$

$$\phi_s(j) = \begin{cases} j - 1 & \text{if } j > s \\ j & \text{otherwise.} \end{cases}$$

The compression operator is only used when  $\Theta_X(s)$  is a prefix of  $\Theta_X(s')$ , and when there is a homomorphism from  $k$  to  $\mathbb{C}_{s,s'}(k)$ . The structure preserving maps associated with the homomorphism are  $\phi_{s,s'}$  and  $\sigma_{\text{id}}$ . Note that the compression operator is defined only when relation  $\prec'$  is asymmetric, and that  $\phi_{s,s'} \circ \phi'_s = \phi_{\text{id}}$ .

**Definition 7.3** (Ordering Enrichment Operator). Operator  $\mathbb{O}$  applied to  $k$  enriches  $\prec_k$  by adding all elements implied by unique origination. That is  $\mathbb{O}(k) = \mathbf{k}_X(\text{rl}, P, \Theta_X, \prec', N, U)$ , where  $k = \mathbf{k}_X(\text{rl}, P, \Theta_X, \prec, N, U)$  and  $\prec' = (\prec \cup \{(n_0, n_1) \mid n_0 \in \mathcal{O}(k, t) \wedge n_1 \in \mathcal{G}(k, t)\})^*$ .

The ordering enrichment operator is total and idempotent. The structure preserving maps associated with the operator's homomorphism are  $\phi_{\text{id}}$  and  $\sigma_{\text{id}}$ , i.e. the homomorphism is an embedding.

**Definition 7.4** (Augmentation Operator). For node  $n$ , role  $r$ , and trace  $C$ , operator  $\mathbb{A}_{n,r,C}$  is:

$$\mathbb{A}_{n,r,C}(\mathbf{k}_X(\text{rl}, P, \Theta_X, \prec, N, U)) = \mathbf{k}_{X'}(\text{rl} \frown r, P, \Theta_X(s) \frown C, \prec', N', U')$$

where  $X'$  is  $X$  extended to include the variables in  $C$ ,  $\prec'$  is the minimal extension of  $\prec$  such that  $(|\Theta_X| + 1, |C|) \prec' n$ ,  $N'$  is  $N$  extended with non-origination assumptions inherited from  $r$  by  $C$ , and likewise for  $U'$ .

The structure preserving maps associated with the augmentation operator's homomorphism are  $\phi_{\text{id}}$  and  $\sigma_{\text{id}}$ , i.e. the homomorphism is an embedding.

# Chapter 8

## Reductions on Preskeletons

This chapter describes the algorithm used to transform a preskeleton into a skeleton as a setwise term reduction system  $(\mathfrak{K}, \rightarrow)$ . Recall that the relation  $k \rightsquigarrow K$  in Chapter 6 is defined in terms of  $\rightarrow \subseteq \mathfrak{K} \times \mathfrak{K}$  by specifying  $\{k\} \rightarrow K$  using  $\rightarrow$ . Additionally, when  $k = k(rl, P, \Theta, \prec, N, U)$ ,  $U_k$  is used to name  $U$ , and so forth for other components of  $k$ .

If a preskeleton  $k$  is not a skeleton, then it is either because some  $t \in U_k$  actually originates at more than one node, or because for some  $t \in U_k$ , there is a node  $n_1 \in \mathcal{G}_k(t)$ , and a node  $n_0 \in \mathcal{O}_k(t)$  such that  $n_0 \not\prec_k n_1$ . These obstructions are resolved via identifying strands and enriching node orderings respectively. A *hulled preskeleton* is a preskeleton in which every uniquely originating atom originates at most once, but it may lack some node orderings needed to be a skeleton.

Skeletons may contain redundant strands. After converting preskeletons to skeletons, a preskeleton reduction system may remove redundant strands using a process called pruning. A skeleton without any redundant strands is called a *pruned skeleton*.

**Definition 8.1** (Hulling Reduction). Preskeleton  $k_0$  reduces to preskeleton  $k_1$  by hulling strands  $s$  and  $s'$ , written  $k_0 \xrightarrow{\mathbb{H}_{s,s'}} k_1$ , iff there is some  $t \in U_{k_0}$  and distinct  $s$  and  $s'$  such that  $\{(s, i), (s', i')\} \subseteq \mathcal{O}_{k_0}(t)$ , a most general unifier  $\sigma$  such that  $\sigma(\Theta_{k_0}(s)(j)) \equiv \sigma(\Theta_{k_0}(s')(j))$  for all  $1 \leq j \leq |\Theta_{k_0}(s)|$ , and  $k_0 \xrightarrow{\mathbb{S}_\sigma} k \xrightarrow{\mathbb{C}_{s,s'}} k_1$  for some  $k$ . For the setwise hulling relation,  $\{k_0\} \xrightarrow{\mathbb{H}_{s,s'}} \{k_1 \mid k_0 \xrightarrow{\mathbb{H}_{s,s'}} k_1\}$ , when  $k_0$  has a message in  $U_{k_0}$  that originates on both  $s$  and  $s'$ .

Note that for  $k_0 \xrightarrow{\mathbb{S}_\sigma} k \xrightarrow{\mathbb{C}_{s,s'}} k_1$ , there is a homomorphism from  $k_0$  to  $k_1$

when there is one from  $k_0$  to  $k$ , i.e. when  $\mathbb{S}_\sigma$  ensures that points of origination are preserved.

**Definition 8.2** (Order Enrichment). Suppose hulled preskeleton  $k_0$  is not a skeleton. Hulled preskeleton  $k_0$  reduces to skeleton  $k_1$  by order enrichment, written  $k_0 \xrightarrow{\mathbb{O}} k_1$ , iff  $k_1$  is the result of adding node orderings implied by origination. That is,  $\prec_{k_1} = (\prec_{k_0} \cup \{(n_0, n_1) \mid n_0 \in \mathcal{O}(k_0, t) \wedge n_1 \in \mathcal{G}(k_0, t)\})^*$ . There is a homomorphism from  $k_0$  to  $k_1$  that is an embedding. For the setwise order enrichment reduction,  $\{k_0\} \xrightarrow{\mathbb{O}} \{k_1 \mid k_0 \xrightarrow{\mathbb{O}} k_1\}$  when  $k_0$  is a hulled preskeleton that is not a skeleton.

The pruning operator  $\mathbb{P}_s$  removes a redundant strand. When a strand  $s$  is redundant, there is another strand  $s'$  that describes more specific behavior. Skeleton  $k_0$  has a redundant strand  $s$  if there is a skeleton  $k_1$  such that  $k_0 \xrightarrow{\mathbb{S}_\sigma} k \xrightarrow{\mathbb{C}_{s,s'}} k_1$ ,  $k_0 \xrightarrow{\phi, \sigma} k_1$ , and  $k_1 \xrightarrow{\phi', \sigma'} k_0$ , where  $\phi \circ \phi' = \phi_{\text{id}}$  and  $\sigma \circ \sigma' = \sigma_{\text{id}}$ .<sup>1</sup> In particular  $\phi = \phi_{s,s'}$  and  $\phi' = \phi'_s$  with  $\phi_{s,s'}$  and  $\phi'_s$  as in Definition 7.2. The bidirectional homomorphism requirement leads to the following definition.

**Definition 8.3** (Pruning). Suppose skeleton  $k_0$  has a redundant strand  $s$ . Then there exists a distinct strand  $s'$  that describes more specific behavior. Skeleton  $k_0$  reduces to skeleton  $k_1$  by pruning, written  $k_0 \xrightarrow{\mathbb{P}_s} k_1$ , iff there is a most general unifier  $\sigma$  such that  $\sigma(\Theta_{k_0}(s)(j)) \equiv \Theta_{k_0}(s')(j)$  for all  $1 \leq j \leq |\Theta_{k_0}(s)|$ , no variable in  $\text{Dom}(\sigma)$  occurs in the trace of any strand other than  $s$ ,  $\sigma$  is a renaming,  $t \in U_{k_0}$  implies  $\sigma(t) \in U_{k_0}$ ,  $t \in N_{k_0}$  implies  $\sigma(t) \in N_{k_0}$ , there is a  $k$  such that  $k_0 \xrightarrow{\mathbb{S}_\sigma} k \xrightarrow{\mathbb{C}_{s,s'}} k_1$ , there is a homomorphism from  $k_0$  to  $k_1$ , and if  $(s, p) \prec_{k_0} (s'', p'')$  then  $(s', p') \prec_{k_0} (s'', p'')$ , and if  $(s'', p'') \prec_{k_0} (s, p)$  then  $(s'', p'') \prec_{k_0} (s', p')$ . For the setwise pruning reduction,  $\{k_0\} \xrightarrow{\mathbb{P}_s} \{k_1 \mid k_0 \xrightarrow{\mathbb{P}_s} k_1\}$ , when there is a  $k_1$  such that  $k_0 \xrightarrow{\mathbb{P}_s} k_1$ .

There is one additional constraint on pruning recently identified. In Chapter 12 on generalization, a parameter to its reduction system is the point-of-view skeleton. The point-of-view skeleton defines the problem to be solved by CPSA. A generalization step is valid only when there is a homomorphism from the point-of-view skeleton and the result of generalization. To support generalization, CPSA computes and stores the strand mapping part of a homomorphism from the point-of-view skeleton to every preskeleton derived from it.

<sup>1</sup>In category theory,  $(\phi', \sigma')$  is a section of  $(\phi, \sigma)$ , and  $(\phi, \sigma)$  is a retraction of  $(\phi', \sigma')$ .

The addition constraint on pruning is this: no strand is considered redundant if it is in the image of the skeleton's strand map from its point-of-view skeleton. Our notation for reductions that include pruning does not record the dependency of the relation on the point-of-view skeleton, but the notation in the chapter on generalization shows how it might be done.

**Definition 8.4** (Thinning). Two strands  $s$  and  $s'$  in a skeleton  $k$  are *effectively equivalent* if  $k$  with  $s$  removed and  $k$  with  $s'$  removed are isomorphic. We can define a different form of pruning based on this idea. Skeleton  $k_0$  reduces to skeleton  $k_1$  by thinning, written  $k_0 \xrightarrow{\mathbb{T}_s} k_1$ , if and only if there exists a strand  $s'$  that is effectively equivalent to  $s$  in  $k_0$  and  $k_1$  is isomorphic to both  $k_0$  with  $s$  removed and  $k_0$  with  $s'$  removed.

Thinning is not used yet.

## 8.1 Reduction Systems

Notice that a setwise hulling reduction may produce the empty set, but a setwise order enrichment and pruning reduction never does.

There are two preskeleton reduction systems, one with pruning, and one without. For the one without, let reduction  $\twoheadrightarrow = \bigcup_{s,s'} \xrightarrow{\mathbb{H}_{s,s'}} \cup \xrightarrow{\mathbb{O}}$ .

**Definition 8.5** (Preskeleton Reduction System). Preskeleton  $k_0$  reduces to skeleton  $k_1$ , written  $k_0 \xrightarrow{skel} k_1$ , if  $\{k_0\} \twoheadrightarrow^* K$ ,  $k_1 \in K$ , and  $K$  is a normal form of  $\twoheadrightarrow$ .

For each skeleton  $k$ ,  $k \xrightarrow{skel} k$ .

When using pruning, let reduction  $\twoheadrightarrow = \bigcup_{s,s'} \xrightarrow{\mathbb{H}_{s,s'}} \cup \xrightarrow{\mathbb{O}} \cup \bigcup_s \xrightarrow{\mathbb{P}_s}$ .

**Definition 8.6** (Preskeleton Reduction System with Pruning). Preskeleton  $k_0$  reduces to pruned skeleton  $k_1$ , written  $k_0 \xrightarrow{pskel} k_1$ , if  $\{k_0\} \twoheadrightarrow^* K$ ,  $k_1 \in K$ , and  $K$  is a normal form of  $\twoheadrightarrow$ .

For each pruned skeleton  $k$ ,  $k \xrightarrow{pskel} k$ .

# Chapter 9

## Penetrator Derivable

For each algebra, the powers of the adversary are defined by a set of roles. For the Basic Crypto Signature in Figure 2.1, the traces of the penetrator roles are in Figure 9.1. Penetrator roles make no origination assumptions.

The context in which penetrator strands appear determine the messages the adversary can derive. The context includes previously sent messages and atoms it is forbidden to originate. An atom that is assumed to be non-originating must be avoided as is a uniquely originating atom that is assumed to originate on a regular strand.

The ternary relation  $T_p : T_a \vdash t$  states that message  $t$  is penetrator derivable from previously sent messages  $T_p$  while avoiding atoms  $T_a$ . The relation is defined by a set of inference rules. Most of the rules are justified by a penetrator role that when instantiated, derives a message in the conclusion of the rule.

The first rule states that no additional penetrator behavior is required to

$$\begin{aligned} \text{base}(t) &= \langle +t \rangle, \text{ where } t \text{ is an atom} \\ \text{tag}(t) &= \langle +t \rangle, \text{ where } t \text{ is a tag} \\ \text{cat}(t_0, t_1) &= \langle -t_0, -t_1, +(t_0, t_1) \rangle \\ \text{sep}(t_0, t_1) &= \langle -(t_0, t_1), +t_0, +t_1 \rangle \\ \text{enc}(t_0, t_1) &= \langle -t_0, -t_1, +\{t_0\}_{t_1} \rangle \\ \text{dec}(t_0, t_1) &= \langle -\{t_0\}_{t_1}, -t_2, +t_0 \rangle, \text{ where } t_2 = \text{inv}(t_1) \\ \text{hash}(t) &= \langle -t, +\#t \rangle \end{aligned}$$

Figure 9.1: Penetrator Traces



derive  $t$  if it has been previously sent.

$$\frac{t \in T_p}{T_p : T_a \vdash t}$$

A uniquely originating atom need not be avoided if it has been sent.

$$\frac{T_p : T_a \vdash t}{\{t_0\} \cup T_p : \{t_0\} \cup T_a \vdash t} \quad (9.1)$$

There are two decomposition steps available to the penetrator.

$$\frac{\{t_0, t_1\} \cup T_p : T_a \vdash t}{\{(t_0, t_1)\} \cup T_p : T_a \vdash t} \quad [\text{by } sep(t_0, t_1)] \quad (9.2)$$

$$\frac{T_p : T_a \vdash inv(t_1) \quad \{t_0, \{\{t_0\}_{t_1}\}\} \cup T_p : T_a \vdash t}{\{\{\{t_0\}_{t_1}\}\} \cup T_p : T_a \vdash t} \quad [\text{by } dec(t_0, t_1)] \quad (9.3)$$

There are three constructive steps.

$$\frac{T_p : T_a \vdash t_0 \quad T_p : T_a \vdash t_1}{T_p : T_a \vdash (t_0, t_1)} \quad [\text{by } cat(t_0, t_1)]$$

$$\frac{T_p : T_a \vdash t_0 \quad T_p : T_a \vdash t_1}{T_p : T_a \vdash \{\{t_0\}_{t_1}\}} \quad [\text{by } enc(t_0, t_1)]$$

$$\frac{T_p : T_a \vdash t}{T_p : T_a \vdash \#t} \quad [\text{by } hash(t)]$$

There are three rules for indivisible messages.

$$T_p : T_a \vdash C_i \quad [\text{by } tag(C_i)]$$

$$\frac{t \notin T_a \quad t \text{ an atom}}{T_p : T_a \vdash t} \quad [\text{by } base(t)]$$

A non-base sorted variable is derivable in a bundle that instantiates it with any message other than an element of  $X_\top$ .

$$\frac{t \in X_\top}{T_p : T_a \vdash t}$$

**Definition 9.1** (Outbound predecessors). The *outbound predecessors* of skeleton  $k$  at  $n$  is  $outpred(k, n) = \{msg_k(n_0) \mid n_0 \prec_k n, n_0 \text{ is transmitting}\}$ .

**Definition 9.2** (Avoidance Set). The *avoidance set* of skeleton  $k$  is  $avoid(k) = N_k \cup \{t \mid t \in U_k \wedge |\mathcal{O}_k(t)| = 1\}$ .

An atom in  $avoid(k)$  is not available to the penetrator, except if it is exposed by a messages transmission. Clearly, only uniquely originating atoms can be exposed.

**Definition 9.3** (Derivable Before). A message  $t$  is *derivable before* reception node  $n$  in skeleton  $k$ , written  $der(k, n, t)$ , if  $T_p : T_a \vdash t$  where  $T_p = outpred(k, n)$  and  $T_a = avoid(k)$ .

**Definition 9.4** (Realized Node). A reception node  $n$  is *realized* in skeleton  $k$  if  $msg_k(n)$  is derivable before  $n$  in  $k$ .

Notice that one can read off penetrator behavior from the proof tree used to demonstrate that  $msg_k(n)$  is derivable before  $n$  in  $k$ . For example, if a decryption step is required by the proof, an instance of the penetrator’s decryption role is indicated. In a bundle, for a non-base sorted variable, there is a substitution that maps the variable to a message that is not a non-base sorted variable. The substitution determines the penetrator behavior associated with the variable.

**Conjecture 9.1** (Realized Skeleton). A skeleton is realized if and only if all of its reception nodes are realized.

*Partial Proof.* Given a skeleton  $k$  in which all of its reception nodes are realized, the combination of the regular behavior in the skeleton, the penetrator behavior specified by the proof trees used to demonstrate each node is realized, and a substitution for non-base sorted variables determines a bundle. The skeleton of the bundle may have more non-originating atoms than is in  $N_k$ , however since the extra non-originating atoms are derivable by the bundle that realizes  $k$ , the proof trees for those atoms specify any additional penetrator behavior required.

The “only if” part of this proof has yet to be completed. □

## 9.1 Implementation

The derivable before a node predicate is implemented using auxiliary functions.

**Definition 9.5** (Buildable). Message  $t$  is *buildable* from previously sent messages  $T_p$  while avoiding  $T_a$ , written  $bld(t, T_p, T_a)$ , if  $T_p : T_a \vdash t$  without the use of Inference Rules 9.1, 9.2, and 9.3.

Consider the following reduction system based on Inference Rules 9.1, 9.2, and 9.3.

$$\begin{array}{ll} \{t\} \cup T_p : T_a \rightarrow T_p : T_a \setminus \{t\} & \text{if } t \text{ is an atom or in } X_{\top} \\ \{(t_0, t_1)\} \cup T_p : T_a \rightarrow \{t_0, t_1\} \cup T_p : T_a & \\ \{\{t_0\}_{t_1}\} \cup T_p : T_a \rightarrow \{t_0, \{\{t_0\}_{t_1}\}\} \cup T_p : T_a & \text{if } t_0 \notin T_p \text{ and} \\ & bld(inv(t_1), T_p, T_a) \end{array}$$

**Definition 9.6** (Decompose). Previously sent messages  $T_p$  and avoidance set  $T_a$  *decompose* to  $T'_p, T'_a$ , written  $decompose(T_p, T_a) = (T'_p, T'_a)$ , if  $T_p : T_a \rightarrow^* T'_p : T'_a$  and  $(T'_p, T'_a)$  is a normal form of reduction  $\rightarrow$ .

The penetrator derivable predicate  $T_p : T_a \vdash t$  is implemented as

$$\begin{array}{l} T_p : T_a \vdash t = \\ \mathbf{let} T'_p, T'_a = decompose(T_p, T_a) \mathbf{in} \\ bld(t, T'_p, T'_a) \end{array}$$

The decomposition at a node function is

$$\begin{array}{l} dcmp(k, n) = \\ decompose(outpred(k, n), avoid(k)) \end{array}$$

The derivable before a node predicate is implemented as

$$\begin{array}{l} der(k, n, t) = \\ \mathbf{let} T_p, T_a = dcmp(k, n) \mathbf{in} \\ bld(t, T_p, T_a) \end{array}$$

# Chapter 10

## Carried Only Within

A set of encryptions  $T_e$  protects critical message  $t$  in message  $t'$  if  $t$  is carried by  $t'$  only within a member of  $T_e$ . The definition of the carried only within (COW) relation to follow makes this concept precise. The concept is used when solving authentication tests (Chapter 11).

**Definition 10.1** (Ancestors). Let  $t' = t @ p$ . The *ancestors* of  $t'$  in  $t$  at  $p$  is the set  $anc(t, p) = \{t @ p' \mid p' \text{ a proper prefix of } p\}$ .

**Definition 10.2** (Carried Only Within). Message  $t$  is *carried only within* set  $T_e$  in  $t'$  if for all carried positions  $p$  of  $t$  in  $t'$ , there exists an ancestor  $t_a \in anc(t', p)$  and  $t_e \in T_e$  such that  $t_a \equiv t_e$ .

The function defines  $carpos(t, t')$  is defined on Page 6, the set of positions at which  $t'$  carries  $t$ . The interface to each algebra exports  $unify_0$ , where

$$unify_0(t, t', \sigma) = \{\sigma' \circ \sigma \mid \sigma' \in unify(\sigma(t), \sigma(t'))\}.$$

The details of a reduction on skeletons called a augmentation will be described in Section 11.2. In simplified form, for an augmentation, given  $t$ ,  $T_e$ , and  $t'$ , one must find all most general unifiers  $\sigma$  such that  $\sigma(t)$  is carried only within set  $\sigma(T_e)$  in  $\sigma(t')$ .

A carried only within solution cannot be directly computed using Definition 10.2. Given terms  $t_a$  and  $t_e$ , the  $unify_0$  function finds substitutions  $\sigma$  such  $\sigma(t_a) \equiv \sigma(t_e)$ , however, the carried positions  $carpos(\sigma(t), \sigma(t'))$ , are used before the  $unify_0$  function computes the substitution  $\sigma$ . Figure 10.1 displays the iterative procedure that breaks the cyclic dependencies. Each step of the iteration improves an approximation of a solution to the problem.

$$\begin{aligned}
cows(t, T, t') &= \\
& cows_0(t, T, t', \sigma_{id}) \quad \text{--- } \sigma_{id} \text{ is the identity subst} \\
cows_0(t, T, t', \sigma) &= \\
& \text{if } \sigma(t) \text{ is COW } \sigma(T) \text{ at } \sigma(t') \text{ then} \\
& \quad \{\sigma\} \\
& \text{else} \\
& \quad \text{let } S = fold(t, T, t', \sigma) \text{ in} \\
& \quad \bigcup_{\sigma' \in S} cows_0(t, T, t', \sigma') \\
fold(t, T, t', \sigma) &= \\
& \{\sigma' \circ \sigma \mid \sigma' \in fold_0(\sigma(T), \sigma(t'), \{\sigma_{id}\}, carpos(\sigma(t), \sigma(t')))\} \\
fold_0(T, t', S, \{\}) &= S \\
fold_0(T, t', S, \langle p \rangle \wedge P) &= \\
& fold_0(T, t', solve(anc(t', p), T, S), P) \\
solve(T, T', S) &= \\
& \{\sigma' \mid t \in T, t' \in T', \sigma \in S, \sigma' \in unify_0(t, t', \sigma)\}
\end{aligned}$$

Figure 10.1: The *cows* Function

$$\begin{aligned}
\mathit{fold}(t, T, t', \sigma) &= \\
&\mathit{fold}_0(t, T, t', \sigma, \{\sigma_{\text{id}}\}, \mathit{carpos}(\sigma(t), \sigma(t'))) \\
\mathit{fold}_0(t, T, t', \sigma, S, \{\}) &= \{\sigma' \circ \sigma \mid \sigma' \in S\} \\
\mathit{fold}_0(t, T, t', \sigma, S, \langle p \rangle \wedge P) &= \\
&\mathit{fold}_0(t, T, t', \sigma, \mathit{solve}(\mathit{anc}(\sigma(t'), p), \sigma(T), S), P)
\end{aligned}$$

Figure 10.2: The Alternate *fold* Function

The correctness of this function is shown in [7], although for a version of the algorithm with the alternate definition for the function *fold* in Figure 10.2.

# Chapter 11

## Solving Authentication Tests

**Definition 11.1** (Protectors). Let  $deriv$  be a boolean valued function that determines if a message is derivable. The encryptions that protect  $t_c$  in  $t$  is  $protectors(deriv, t_c, t) = prot(t)$  where

$$prot(t) = \begin{cases} \text{undefined} & \text{if } t \equiv t_c, \text{ else} \\ \{\} & \text{if } t = \{\{t_0\}\}_{t_1} \text{ and } t_c \text{ is not carried by } t_0, \text{ else} \\ \{\{t_0\}\}_{t_1} & \text{if } t = \{\{t_0\}\}_{t_1} \text{ and } \neg deriv(inv(t_1)), \text{ else} \\ prot(t_0) & \text{if } t = \{\{t_0\}\}_{t_1}, \text{ else} \\ \bigcup_{i < n} prot(t_i) & \text{if } t = f(t_0, \dots, t_{n-1}) \text{ and } t \text{ is not an atom, else} \\ \{\} & \text{otherwise.} \end{cases}$$

**Definition 11.2** (Escape Set). The *escape set* for message  $t_c$  at  $n$  in skeleton  $k$  is the set of encryptions  $esc(k, n, t_c)$  where

$$esc(k, n, t_c) = \{t_e \mid t_e \in protectors(\lambda t. der(k, n, t), t_c, t_o), t_o \in outpred(k, n)\}$$

and  $der(k, n, t)$  is true when  $t$  is derivable before  $n$  in  $k$  (See Definition 9.3).

The  $der$  function is implemented as  $der(k, n, t) = bld(t, T_p, T_a)$  where  $(T_p, T_a) = dcmp(k, n)$ , so that  $T_p$  and  $T_a$  need not be recomputed.

**Definition 11.3** (Critical Position). Position  $p$  is a *critical position* of  $t = msg_k(n)$  if

1.  $p$  is a carried position in  $t$ ,
2. either  $t @ p \in U_k$ ,  $t @ p = \#t_1$ , or  $t @ p = \{\{t_0\}\}_{t_1}$  and  $t_1$  is not derivable before  $n$  in  $k$ ,

3.  $esc(k, n, t @ p)$  is defined, and
4.  $anc(t, p) \cap esc(k, n, t @ p) = \emptyset$ .

The message at a critical position is called an *authentication test*. It is a *nonce test* if the message is an atom, otherwise it is an *encryption test*. (A hash is treated as a kind of encryption in which the term that is hashed is the encryption key.) Observe that every critical message at a node in a skeleton is not derivable at the node.

**Conjecture 11.1.** A reception node is unrealized iff it has a critical position.

**Definition 11.4** (Target Messages). Let  $T_e$  be a set of messages, and  $t_c$  be a message. The set of *target messages* is

$$targ(t_c, T_e) = \{t_c\} \cup \{t_t \mid t_e \in T_e, p \in carpos(t_c, t_e), t_t \in anc(t_e, p)\} \setminus T_e.$$

**Definition 11.5** (Critical Position Solved). Suppose  $p$  is a critical position at  $n_0$  in  $k_0$  and  $k_0 \xrightarrow{\phi, \sigma} k_1$ . Let  $t_0 = msg_k(n) @ p$ ,  $t_1 = \sigma(t_0)$ ,  $T_0 = esc(k_0, n_0, t_0)$ ,  $T_1 = \sigma(T_0)$ ,  $n_1 = \phi(n_0)$ , and  $t = msg_{k_1}(n_1)$ . Critical position  $p$  is *solved* in  $k_1$  after  $k_0$  at  $n_0$  if:

1.  $anc(t, p) \cap T_1 \neq \emptyset$ , or
2. for some  $t_p \in outpred(k_1, n_1)$ ,  $t_1$  is not carried only within  $T_1$  in  $t_p$ , or
3.  $targ(t_1, esc(k_1, n_1, t_1)) \setminus \sigma(targ(t_0, T_0)) \neq \emptyset$  and there are variables in  $k$ 's protocol that are not atoms, or
4. the decryption key of a member of  $T_1$  is derivable before  $n_1$  in  $k_1$ , or
5.  $t_1$  is an encryption and its encryption key is derivable before  $n_1$  in  $k_1$ .

## 11.1 Test Solving Steps

A step used to solve a test is a contraction (Definition 11.6), a regular augmentation (Definition 11.7), a displacement (Definition 11.8), or a listener augmentation (Definition 11.9).



**Definition 11.6** (Contraction). Let  $p$  be a critical position at  $n$  in  $k$ ,  $t = msg_k(n)$ , and  $T_e = esc(k, n, t @ p)$ . Suppose there is a substitution  $\sigma$  such that for some  $t_a \in anc(t, p)$ ,  $t_e \in T_e$ ,  $\sigma(t_a) = \sigma(t_e)$ . Skeleton  $k_1$  is a *contraction* if  $k \xrightarrow{\mathbb{S}_\sigma} k_0 \xrightarrow{pskel} k_1$ .

CPSA computes a set of substitutions for each critical position, and then removes some substitutions to form a complete set of most general unifiers. Only most general unifiers are used for contractions.

**Definition 11.7** (Regular Augmentation). Suppose substitution  $\sigma$ , non-listener role  $r$ , and trace  $C$  are selected as described in Section 11.2. Skeleton  $k_2$  is a *regular augmentation* if  $k \xrightarrow{\mathbb{S}_\sigma} k_0 \xrightarrow{\mathbb{A}_{n,r,C}} k_1 \xrightarrow{pskel} k_2$ .

**Definition 11.8** (Displacement). Let substitution  $\sigma$ , non-listener role  $r$ , and trace  $C$  be selected as described in Section 11.2, and there be a preskeleton  $k_1$  such that  $k \xrightarrow{\mathbb{S}_\sigma} k_0 \xrightarrow{\mathbb{A}_{n,r,C}} k_1$ . Similar to a hulling reduction (Definition 8.1), suppose there are strands  $s$  and  $s'$ , where one of them is the newly created strand, and a most general unifier  $\sigma'$  such that  $\sigma'(\Theta_{k_1}(s)(j)) \equiv \sigma'(\Theta_{k_1}(s')(j))$  for  $1 \leq j \leq |\Theta_{k_1}(s)|$ . Skeleton  $k_4$  is a *displacement* if  $k_1 \xrightarrow{\mathbb{S}_{\sigma'}} k_2 \xrightarrow{\mathbb{C}_{s,s'}} k_3 \xrightarrow{pskel} k_4$ .

**Definition 11.9** (Listener Augmentation). Let  $p$  be a critical position at  $n$  in  $k$ ,  $t_c = msg_k(n) @ p$ , and  $T_e = esc(k, n, t_c)$ . For each  $\{t_0\}_{t_1} \in T_e$ , skeleton  $k_1$  is a *listener augmentation* if  $k \xrightarrow{\mathbb{A}_{n,lsn,C}} k_0 \xrightarrow{pskel} k_1$  and  $C$  listens for  $inv(t_1)$ , i.e.  $C = \langle -inv(t_1), +inv(t_1) \rangle$ . If  $t_c = \{t_0\}_{t_1}$ , then skeleton  $k_1$  is a *listener augmentation* if  $k \xrightarrow{\mathbb{A}_{n,lsn,C}} k_0 \xrightarrow{pskel} k_1$  and  $C = \langle -t_1, +t_1 \rangle$ .

**Definition 11.10** (Cohort Member). For unrealized node  $n$  in a skeleton  $k_0$ , and a position  $p$  at  $n$ ,  $k_0 \xrightarrow{n,p} k_1$  asserts that  $k_1$  is a member of the cohort of  $k_0$ , where  $k_1$  is derived using contraction, regular augmentation, displacement, or listener augmentation, and  $p$  is solved in  $k_1$  after  $k_0$  at  $n$ . For the setwise cohort member reduction,  $\{k_0\} \xrightarrow{n,p} \{k_1 \mid k_0 \xrightarrow{n,p} k_1\}$ , when  $n$  is unrealized in  $k_0$ , and  $p$  is a critical position at  $n$ .

**Conjecture 11.2** (Critical Message Solved). If  $k_0 \xrightarrow{n_0,p_0} k_1 \xrightarrow{n_1,p_1} \dots \xrightarrow{n_{\ell-1},p_{\ell-1}} k_\ell$  is a sequence of cohort member reductions, then for positive  $\ell$ ,  $p_0$  is solved in  $k_\ell$  after  $k_0$  at  $n_0$ .

## 11.2 Augmentation

Let  $t_c$  be the critical message that demonstrates  $n$  is a test node in skeleton  $k$ . For each triple  $(\sigma, r, C)$  that satisfies some properties, there is a potential regular augmentation with  $\xrightarrow{pskel} \circ \xrightarrow{\mathbb{A}_{n,r,C}} \circ \xrightarrow{\mathbb{S}_\sigma}$ . When successful, the message  $t$  in the last event of the added strand is outbound, carries  $\sigma(t_c)$ , but  $\sigma(t_c)$  is not carried only within escape set  $\sigma(T_e)$  in  $t$ , where  $T_e = \text{esc}(k, n, t_c)$ , the escape set. Moreover, for every other message  $t$  in the strand,  $\sigma(t_c)$  is carried only within escape set  $\sigma(T_e)$  in  $t$ . The last event in the strand is called a *transforming event*, as this event no longer protects the critical message, but events that precede it do.

CPSA computes the parameters for a set of augmentation steps as follows. Suppose skeleton  $k = \mathbf{k}_X(-, P, -, -, -)$ . First, compute the target messages,  $T_t = \text{targ}(t_c, T_e)$ . Next, for each non-listener role  $r(C_r, N_r, U_r) \in P$  and each index  $h$  where  $C_r(h) = +t$ , a transmission, do the following.

**Create fresh variables:** Let  $\sigma_r$  be a sort preserving variable renaming, where the domain is the variables that occur in  $C_r|_h$ , and every variable in the range does not occur in  $X$  or in  $P$ .

**Insert critical message:** For each message  $t'$  carried by  $t$ , and each  $t_t \in \text{targ}(t_c, T_e)$ , consider most general unifiers  $\sigma'$  where,  $\sigma'(t') = \sigma'(t_t)$  and  $\sigma_r \trianglelefteq \sigma'$ . (In other words,  $\sigma' = \sigma_0 \circ \sigma_r$  for some  $\sigma_0$ .)

**Ensure previous events do not transform:** For each  $\sigma'$ , find most general unifiers  $\sigma$  such that for  $1 \leq i < h$ ,  $\sigma(t_c)$  is carried only within  $\sigma(T_e)$  at  $\sigma(C(i))$  and  $\sigma' \trianglelefteq \sigma$ . The function *cowt*, presented in Figure 11.1, performs the explorations, producing the substitutions  $S' = \text{cowt}(t_c, T_e, C_r|_h, S)$ . Function *fold* is defined in Figure 10.1. Let  $S_{r,h}$  be the set  $S'$  with non-most general unifiers removed.

**Ensure last event transforms:** For each  $\sigma \in S_{r,h}$ , if  $\sigma(t_c)$  is not carried only with  $\sigma(T_e)$  at  $\sigma(C(h))$ , try augmenting with parameters  $n$ ,  $r$ ,  $\sigma \circ C|_h$ , and  $\sigma$ .

*For target terms to be the reasonable set for insertion of the critical message, one must require that variables of sort message are acquired. This fact needs to be explained and noted as another reason for the acquired variable constraint.*

$$\begin{aligned}
\text{cowt}(t, T, C, S) &= \\
&\bigcup_{\sigma \in S} \text{cowt}_0(t, T, C, \sigma) \\
\text{cowt}_0(t, T, C, \sigma) &= \\
&\text{if } \forall t \pm t' \in C \rightarrow \sigma(t) \text{ is COW } \sigma(T) \text{ at } \sigma(t') \text{ then} \\
&\quad \{\sigma\} \\
&\text{else} \\
&\quad \text{cowt}(t, T, C, \text{foldn}(t, T, C, \{\sigma\})) \\
\text{foldn}(t, T, \langle \rangle, S) &= S \\
\text{foldn}(t, T, \langle \pm t' \rangle \wedge C, S) &= \\
&\text{foldn}(t, T, C, \bigcup_{\sigma \in S} \text{fold}(t, T, t', \sigma))
\end{aligned}$$

Figure 11.1: The *cowt* Function

# Chapter 12

## Generalization

Each problem statement for CPSA is expressed as a preskeleton. If the preskeleton cannot be transformed into a single skeleton using the Preskeleton Reduction System, an error is signaled. Otherwise, the first skeleton is designated as the point-of-view skeleton. For each skeleton generated from a point-of-view skeleton via contraction, augmentation, and generalization, there is a homomorphism from the point-of-view skeleton. Simplifying the implementation is the motivation for restricting the algorithm to problem statements that are expressed by a single skeleton.

**Definition 12.1** (Generalize). A skeleton  $k_0$  *generalizes* skeleton  $k_1$ , written  $k_1 \xrightarrow{\leq}_k k_0$ , if both  $k_0$  and  $k_1$  are realized,  $k_0$  and  $k_1$  are not isomorphic, there is a homomorphism from a point-of-view skeleton  $k$  to  $k_0$ , and a strandwise injective homomorphism  $k_0 \mapsto k_1$ .

If skeletons are allowed to be isomorphic, we write  $k_1 \xrightarrow{\leq}_k k_0$ , and note that  $\xrightarrow{\leq}_k$  defines a partial ordering. Therefore, there are maximal elements in the partial ordering. A shape associated with a preskeleton is a maximally generalized realized skeleton derived from the preskeleton.

We write  $k = pov(k_0)$  if there is a unique  $k$  such that  $k_0 \xrightarrow{skel} k$  and there is a homomorphism from  $k_0$  to  $k$ .

**Definition 12.2** (Shape). Let  $k_0$  be a preskeleton such that  $k = pov(k_0)$ , and let  $k_1$  be a realized skeleton such that  $k \mapsto k_1$ . Skeleton  $k_2$  is a *shape* of  $k_0$  if  $k_1 \xrightarrow{\leq}_k k_2$ , and  $k_2$  is maximal among skeletons that generalize  $k_1$ .

There are four generalization reductions used to transform a realized skeleton into its shapes: deletion, weakening, forgetting, and separation.

**Definition 12.3** (Deletion). Skeleton  $k_0$  *generalizes by deletion* skeleton  $k_1$ , written  $k_1 \xrightarrow{\mathbb{D}_n}_k k_0$ , if  $k_1 \xrightarrow{<}_k k_0$ ,  $k_2 \xrightarrow{skel}_k k_0$ , and  $k_2$  is the result of deleting node  $n$  in  $k_1$  and all of the nodes that follow it in its strand.

**Definition 12.4** (Weakening). Skeleton  $k_0$  *generalizes by weakening* skeleton  $k_1$ , written  $k_1 \xrightarrow{\mathbb{W}_{n,n'}}_k k_0$ , if  $k_1 \xrightarrow{<}_k k_0$ ,  $k_2 \xrightarrow{skel}_k k_0$ , and  $k_2$  is  $k_1$  except  $\prec_{k_2} = (\prec_{k_1} \setminus \{(n, n')\})^*$ .

**Definition 12.5** (Forgetting). Skeleton  $k_0$  *generalizes by origination assumption forgetting* skeleton  $k_1$ , written  $k_1 \xrightarrow{\mathbb{F}_t}_k k_0$ , if  $k_1 \xrightarrow{<}_k k_0$ ,  $k_2 \xrightarrow{skel}_k k_0$ , and  $k_2$  is  $k_1$  except  $U_{k_2} = U_{k_1} \setminus \{t\}$  and  $N_{k_2} = N_{k_1} \setminus \{t\}$ .

Sometimes a more general skeleton can be found by replacing some occurrences of one variable by a fresh variable. For variable separation, the location of an occurrence of a variable is defined using a skeleton's instance sequence.

**Definition 12.6** (Location). Message  $t$  is at *location*  $(s, i, p)$  in  $k$  if  $\pm t = \Theta_k(s)(i) @ p$ .

**Definition 12.7** (Variable Separation). Skeleton  $k_0$  *generalizes by variable separation* skeleton  $k_1$ , written  $k_1 \xrightarrow{\mathbb{V}_t}_k k_0$ , if  $k_1 \xrightarrow{<}_k k_0$ ,  $k_2 \xrightarrow{skel}_k k_0$ , and  $k_2$  is  $k_1$  except  $t$  is a variable that occurs in multiple locations in  $k_1$ , and  $k_2$  is the result of replacing  $t$  with a variable  $t_0$  of the same sort at a subset of  $t$ 's locations, where  $t_0$  occurs nowhere in  $k_1$ .

When separating a non-originating term, both the term and its clone are non-originating. When separating a uniquely originating term, either the term or its clone is uniquely originating.

*What happens when separating  $t$  in  $k$  into  $t$  and  $t_0$ , and  $\text{ltk}(t, t) \in N_k$ ? Should a skeleton  $k_0$  with  $\text{ltk}(t, t_0) \in N_{k_0}$  be a candidate separation? Currently, only skeletons  $k_1$  with  $\text{ltk}(t, t) \in N_{k_1}$  and  $\text{ltk}(t_0, t_0) \in N_{k_1}$  are considered.*

**Definition 12.8** (Generalization). The reduction  $\xrightarrow{gen}_k = \bigcup_n \xrightarrow{\mathbb{D}_n}_k \cup \bigcup_{n,n'} \xrightarrow{\mathbb{W}_{n,n'}}_k \cup \bigcup_t \xrightarrow{\mathbb{F}_t}_k \cup \bigcup_t \xrightarrow{\mathbb{V}_t}_k$  is the *generalization* relation. For the setwise generalization reduction,  $\{k_0\} \xrightarrow{gen}_k \{k_1\}$  when  $k_0 \xrightarrow{gen}_k k_1$ .

*The fact that each generalization reduction replaces a singleton with just a singleton requires explanation. We're not sure it's justified, and harbor serious doubts. It is justified if we can prove the conjecture that the cohort reduction relation produces every shape, and generalization just identifies which realized skeletons are shapes. However, one member of the test suite provides a counter example to the conjecture. Barring a bug in CPSA, the conjecture must be false.*

**Conjecture 12.1** (Generalization). The relation  $\xrightarrow{k}^{gen}$  is terminating.

## Discussion

In [4], the shapes of a point-of-view skeleton are said to be minimal, in the partial ordering induced by injective homomorphism, among all realized homomorphic images of the point-of-view skeleton. Minimal corresponds to maximally generalized. The need for origination assumption forgetting was not known when [4] was written. Generalization by variable separation uses non-carried positions, and in particular, positions that traverse an atom edge. Algebras in previous strand space papers have no concept of a position that traverses an atom edge, and therefore cannot be used to specify generalization by variable separation.

# Chapter 13

## Collapsing

Let reduction  $\xrightarrow{cg}_k = (\xrightarrow{co} \cup \xrightarrow{gen})^+$ , the transitive closure of the cohort and generalization setwise reduction relations. The normal forms of this relation are sets of shapes, however, shapes may be missing from each set. This occurs when CPSA is run in a mode that omits displacements when solving authentication tests, which is sometimes done for performance reasons. The missing shapes are found by collapsing other shapes.

**Definition 13.1** (Collapsing). Let  $k_0$  and  $k_1$  be two skeletons such that there are two strands,  $s$  and  $s'$ , and a most general unifier  $\sigma$  such that  $\sigma(\Theta_{k_0}(s)(j)) \equiv \sigma(\Theta_{k_0}(s')(j))$  for all  $1 \leq j \leq |\Theta_{k_0}(s)|$ . Then  $k_0$  *collapses* to  $k_1$ , written  $k_0 \xrightarrow{clp} k_1$ , if  $k_0 \xrightarrow{\mathbb{S}_\sigma} k \xrightarrow{\mathbb{C}_{s,s'}} k' \xrightarrow{skel} k_1$ .

**Definition 13.2** (Setwise Collapsing). For the collapsing relation  $\xrightarrow{clp}_k \subseteq 2^{\mathfrak{R}} \times 2^{\mathfrak{R}}$ ,  $K_0 \xrightarrow{clp}_k K_1$  if  $K_0$  is a normal form of  $\xrightarrow{cg}_k$ , for some  $k_0 \in K_0$ ,  $k_0 \xrightarrow{clp} k$ ,  $\{k\} \cup K_0 \xrightarrow{cg}_k K_1$ , and  $K_0 \neq K_1$ .

Notice that a setwise cohort reduction may produce the empty set, but a setwise generalization and collapsing reduction never does.

# Chapter 14

## Skeleton Reduction System

Let reduction  $\rightarrow_k = \xrightarrow{cg}_k \cup (\xrightarrow{clp}_k)^+$ . This reduction system specifies the CPSA program.

**Conjecture 14.1.** The reduction  $\rightarrow_k$  is confluent.

**Conjecture 14.2** (Soundness). Let  $k_0$  be a preskeleton and  $k$  be an unrealized skeleton such that  $k = pov(k_0)$ . Skeleton  $k_1$  is a shape of  $k_0$  if  $\{k\} \rightarrow_k K$ ,  $k_1 \in K$ , and  $K$  is a normal form.

The set of bundles denoted by preskeleton  $k$ ,  $\llbracket k \rrbracket$  is defined on Page 13.

**Conjecture 14.3** (Completeness). Let  $k_0$  be a preskeleton and  $k$  be an unrealized skeleton such that  $k = pov(k_0)$ . For all  $K$  such that  $\{k\} \rightarrow_k K$ ,  $\llbracket k_0 \rrbracket = \bigcup_{k_1 \in K} \llbracket k_1 \rrbracket$ .



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Carolyn Talcott and Leonard Monk provided valuable feedback on drafts of this document.

# Appendix A

## Programs Specified by a Role

Given the definitions in Chapter 3, a role can be viewed as an abstraction of a program, and a strand as an abstraction of a run of a program. But what program is specified by a role?

Consider a role that contains the event  $-\{t_0\}_{t_1}$ . If the program has the decryption key  $inv(t_1)$  before the message is received, the program could decrypt the message and extract  $t_0$ . Alternatively, if the program has  $\{t_0\}_{t_1}$ , or has  $t_0$  and the encryption key  $t_1$ , it might check to see if the received message is the same as the expected message, and abort the run if not.

Here is an example of when the second behavior is desired. In CPSA, before hashing was part of the algebra, an encryption was used to represent hashing. The hash of  $t$ ,  $\#t$ , expanded to  $\{\text{“hash”}, t\}_h$ , where  $h$  was an asymmetric key known to all, but no one knew  $h^{-1}$ . The tag “hash” was added to the encryption so as to ensure a hash was never confused with other uses of encryption.

A role does not specify a valid program if the only possible way of interpreting the event  $-\#t$  is by using  $h^{-1}$  to decrypt  $\{\text{“hash”}, t\}_h$ . The remainder of this section describes how this class of specification errors is detected for the Basic Crypto Algebra.

The behaviors associated with a trace depend on the set of messages available initially. The behaviors are specified by a data flow relation,  $T_0, C \triangleright T_1$ . For trace  $C$ , the relation  $T_0, C \triangleright T_1$  asserts that when messages  $T_0$  are available initially, there is a behavior of  $C$  that produces messages  $T_1$ .

A derivation tree used to demonstrate  $T_0, C \triangleright T_1$  shows the steps that enable the flow of data. The tree can be linearized, and thus specifies a sequential program that implements the role.

The data flow relation is defined with the aid of a data flow relation for a sequence of events,  $T_0, C \triangleright T_1$ .

$$T, \langle \rangle \triangleright T \quad \frac{T_0, \langle \pm t \rangle \triangleright T \quad T, C \triangleright T_1}{T_0, \langle \pm t \rangle \wedge C \triangleright T_1}$$

The  $T_0, \langle \pm t \rangle \triangleright T_1$  relation is defined using the  $T_0, C \triangleright T_1$  relation. An outbound message can be formed if it is available initially

$$\frac{t \in T}{T, \langle +t \rangle \triangleright T}$$

or if it can be formed by construction.

$$\frac{T, \langle +t_1, \dots, +t_n \rangle \triangleright T}{T, \langle +f(t_1, \dots, t_n) \rangle \triangleright T} \quad \left[ \begin{array}{l} f(t_1, \dots, t_n) \\ \text{not an atom} \end{array} \right]$$

An inbound message makes atoms and acquired variables available.

$$T, \langle -t \rangle \triangleright T \cup \{t\} \quad [t \text{ an atom or a variable}]$$

When the decryption key is available, the contents of the encryption are also available. Furthermore, the encryption can be sent in future messages without access to its encryption key.

$$\frac{T_0, \langle +inv(t_1) \rangle \triangleright T_0 \quad T_0, \langle -t_0 \rangle \triangleright T_1}{T_0, \langle -\{t_0\}_{t_1} \rangle \triangleright T_1 \cup \{\{t_0\}_{t_1}\}}$$

A received encryption that can be sent ensures the encryption agrees with currently available terms and makes nothing new available.

$$\frac{T, \langle +\{t_0\}_{t_1} \rangle \triangleright T}{T, \langle -\{t_0\}_{t_1} \rangle \triangleright T}$$

Consider an operation  $f$  other than the encryption operation. The order in which messages that occur in a message constructed using  $f$  are made available may determine if the decryption key of an encryption is available. All possible orderings must be explored. Let  $\pi_n$  be a permutation on the domain of a sequence of length  $n$ .

$$\frac{T_0, \langle -t_1, \dots, -t_n \rangle \circ \pi_n \triangleright T_1}{T_0, \langle -f(t_1, \dots, t_n) \rangle \triangleright T_1} \quad \left[ \begin{array}{l} f(t_1, \dots, t_n) \\ \text{not an atom} \end{array} \right]$$

The data flow relation is used to find initial sets of atoms that are compatible with some behavior of a trace that produces messages.

|   |  |
|---|--|
| <pre> proc(<math>a, n, K_b, K_a^{-1}</math>)   send(<math>\{a, n\}_{K_b}</math>);   <math>x_0 \leftarrow \text{recv}()</math>;   <math>x_1 \leftarrow \text{decrypt}(x_0, K_a^{-1})</math>;   <math>x_1 \neq n \rightarrow \text{fail}</math>; end </pre> | <pre> proc(<math>a, n, K_b, K_a</math>)   send(<math>\{a, n\}_{K_b}</math>);   <math>x_0 \leftarrow \text{recv}()</math>;   <math>x_1 \leftarrow \{n\}_{K_a}</math>;   <math>x_0 \neq x_1 \rightarrow \text{fail}</math>; end </pre> |
|---|--|

Figure A.1: Two programs that implement the role  $\langle +\{a, n\}_{K_b}, -\{n\}_{K_a} \rangle$ .

**Definition A.1** (Trace Parameters). The set of atoms  $T_0$  are *parameters* of trace  $C$  if  $T_0, C \triangleright T_1$  for some  $T_1$ , and  $T_0$  is minimal, that is for all  $T'_0$  such that  $T'_0, C \triangleright T_1$ ,  $T'_0 \not\subseteq T_0$ .

The role  $\langle +\{a, n\}_{K_b}, -\{n\}_{K_a} \rangle$  has two sets of parameters,  $\{a, n, K_b, K_a^{-1}\}$  and  $\{a, n, K_b, K_a\}$ . See Figure A for two examples of programs that implement the role, using distinct parameter sets.

The CPSA distribution contains a program that computes the set of parameters of a role. It was used to find an error in a role's use of hashing as described at the beginning of this section. The role in question is the verifier role that is part of an attestation protocol [2]. In an earlier version of the role, every set of parameters included the decryption key of the encryption used as a hash. The role was so complicated that inspection did not reveal the error.

Note that this section is specific to the basic crypto algebra in the sense that all non-atomic operations are assumed to be constructable, and encryption has the specific de-construction recipe shown. Although carried positions were not mentioned in this section, the inference system specifies the same concept. In fact, in future work we hope to show that the inference system is all one needs to define, and that which positions are carried, which are protected, and which sorts are atoms can be defined in terms of the inferences available.

# Appendix B

## Shape Analysis and First-Order Logic

For each point-of-view skeleton and its shapes found by CPSA, there is a formula in the language of order-sorted first-order logic called a *shape analysis sentence*, often shortened to a shape sentence. The sentence has a special form,  $\forall X(\Psi \supset \bigvee_i \exists Y_i(\Delta_i \wedge \Phi_i))$ , where  $\Psi$  and  $\Phi_i$  are conjunctions of atomic formulas and  $X$  and  $Y_i$  are variable sets. This fragment of first-order logic is called coherent logic. Formula  $\Psi$  describes the point-of-view skeleton  $k_0$ . For each homomorphism to a shape,  $k_0 \xrightarrow{\delta_i} k_i$ , formula  $\Delta_i$  describes the structure preserving maps  $\delta_i$ , and the shape  $k_i$  is described by  $\Phi_i$ .

An interpretation of a shape sentence is a skeleton. If CPSA finds all of the shapes and the homomorphisms associated with a point-of-view skeleton, the analysis' shape sentence is satisfied in all realized skeletons. Shape sentences are closely related to security goals [6], and motivated by that work.

### B.1 Shape Formulas

The signature for terms extends the one used for the underlying message algebra with a sort **nat**, the sort of natural numbers, and two new operations, constant **zero**: **nat**, and the successor function **succ**: **nat**  $\rightarrow$  **nat**. The text uses the usual numerals for natural numbers. Variables of this sort will range over strands.

Shape formulas make use of protocol specific predicates and protocol independent predicates. For each role  $r = r_Y(C, N, U)$  in protocol  $P$ , there are

protocol specific binary predicates  $P[r, h, x]: \mathbf{nat} \times S$  for every  $1 \leq h \leq |C|$  and  $x: \mathbf{S}$  that occurs in  $C|_h$ . The protocol independent predicate of arity four is **prec**:  $\mathbf{nat} \times \mathbf{nat} \times \mathbf{nat} \times \mathbf{nat}$ . The protocol independent unary predicates are **non**:  $B$  and **uniq**:  $B$  for each base sort  $B$ , and the protocol independent ternary predicates are **orig**:  $B \times \mathbf{nat} \times \mathbf{nat}$ . The predicate **false** has arity zero and, of course, equality is binary.

We define  $\mathcal{F}(k) = (Y, \Phi)$ , where  $\Phi$  is  $k$ 's skeleton formula, and  $Y$  is the formula's variable set. Using the external syntax presented in Section 5.4, let  $k = k_X(P, I, \prec, N, U)$ . The variable set  $Y$  is  $X$  augmented with a variable  $z_s: \mathbf{nat}$  for each strand  $s \in \text{Dom}(I)$ . The formula  $\Phi$  is a conjunction of atomic formulas composed as follows.

- For each  $s \in \text{Dom}(I)$ , let  $I(s) = i(r, h, \sigma)$ . For each variable  $x \in \text{Dom}(\sigma)$  and term  $t = \sigma(x)$ , assert  $P[r, h, x](z_s, t)$ .
- For each  $(s, i) \prec (s', i')$ , assert **prec**( $z_s, i, z_{s'}, i'$ ).
- For each  $t \in N$ , assert **non**( $t$ ).
- For each  $t \in U$ , assert **uniq**( $t$ ).
- For each  $t \in U$  and  $(s, i) \in \mathcal{O}_k(t)$ , assert **orig**( $t, z_s, i$ ).

In the code that extracts a shape analysis sentence, the **prec** predicate is not asserted for strand succession, and only for communication when it is in the transitive reduction of the  $\prec$  relation. The missing relations must be asserted as axioms for proper handling of a shape sentence.

Given a set of homomorphisms  $k_0 \xrightarrow{\delta_i} k_i$ , its shape sentence is

$$\mathcal{S}(k_0 \xrightarrow{\delta_i} k_i) = \forall X(\Psi \supset \bigvee_i \exists Y_i(\Delta_i \wedge \Phi_i)), \quad (\text{B.1})$$

where  $\mathcal{F}(k_0) = (X, \Psi)$ . The same procedure produces  $Y_i$  and  $\Phi_i$  for shape  $k_i$  with one proviso—the variables in  $Y_i$  that also occur in  $X$  must be renamed to avoid trouble while encoding the structure preserving maps  $\delta_i$ .

The structure preserving maps  $\delta_i = (\phi_i, \sigma_i)$  are encoded in  $\Delta_i$  by a conjunction of equalities. Map  $\sigma_i$  is coded as equalities between a message algebra variable in the domain of  $\sigma_i$  and the term it maps to. Map  $\phi_i$  is coded as equalities between strand variables in  $\Psi$  and strand variables in  $\Phi_i$ . Let  $Z$  be the sequence of strand variables freshly generated for  $k_0$ ,

$$\begin{aligned}
& \forall a_0, b_0 : \mathbf{A}, s_0 : \mathbf{S}, d_0 : \mathbf{D}, z_0 : \mathbf{N} ( \\
& \quad \text{init}_{2,a}(z_0, a_0) \wedge \text{init}_{2,b}(z_0, b_0) \wedge \text{init}_{2,s}(z_0, s_0) \wedge \text{init}_{2,d}(z_0, d_0) \wedge \\
& \quad \text{non}(a_0^{-1}) \wedge \text{non}(b_0^{-1}) \wedge \text{uniq}(s_0) \wedge \text{orig}(s_0, z_0, 1) \\
& \quad \supset \\
& \quad \exists a_1, b_1 : \mathbf{A}, s_1 : \mathbf{S}, d_1 : \mathbf{D}, z_1, z_2 : \mathbf{N} ( \\
& \quad \quad z_0 = z_1 \wedge a_0 = a_1 \wedge b_0 = b_1 \wedge s_0 = s_1 \wedge d_0 = d_1 \wedge \\
& \quad \quad \text{init}_{2,a}(z_1, a_1) \wedge \text{init}_{2,b}(z_1, b_1) \wedge \text{init}_{2,s}(z_1, s_1) \wedge \text{init}_{2,d}(z_1, d_1) \wedge \\
& \quad \quad \text{resp}_{2,a}(z_1, a_1) \wedge \text{resp}_{2,b}(z_1, b_1) \wedge \text{resp}_{2,s}(z_1, s_1) \wedge \text{resp}_{2,d}(z_1, d_1) \wedge \\
& \quad \quad \text{prec}(z_1, 1, z_2, 1) \wedge \text{prec}(z_2, 2, z_1, 2) \wedge \\
& \quad \quad \text{non}(a_1^{-1}) \wedge \text{non}(b_1^{-1}) \wedge \text{uniq}(s_1) \wedge \text{orig}(s_1, z_1, 1))
\end{aligned}$$

Figure B.1: A Shape Analysis Sentence for Blanchet's Protocol

and  $Z_i$  be the ones generated for  $k_i$ . The strand mapping part of  $\Delta_i$  is  $\bigwedge_{j \in \text{Dom}(\Theta)} Z(j) = Z_i(\phi_i(j))$ .

The shape analysis sentence for the first analysis of Blanchet's Simple Example Protocol in Section 5.1 is displayed in Figure B.1. The sort **nat** is abbreviated as  $\mathbf{N}$ , and the strand progress predicate  $P[r, h, x](z, t)$  is written  $r_{h,x}(z, t)$  with the protocol left implicit.

## B.2 Semantics of Shape Formulas

Let  $k = k_X(rl, P, \Theta, \prec, N, U)$ . The universe of discourse is  $\mathfrak{D} = \mathbf{N} \cup \mathfrak{A}_X$ . When formula  $\Psi$  is satisfied in skeleton  $k$  with variable assignment  $\alpha : Y \rightarrow \mathfrak{D}$ , we write  $k, \alpha \models \Psi$ . When sentence  $\Sigma$  is satisfied in skeleton  $k$ , we write  $k \models \Sigma$ .

For each protocol specific predicate  $P[r, h, x]$ ,  $k, \alpha \models P[r, h, x](y, z)$  iff  $\alpha(y) \in \mathbf{N}$ ,  $\alpha(z) \in \mathfrak{A}$ , and with  $\alpha(y) = s$  and  $r = r(C, N, U)$ ,

1.  $s \in \text{Dom}(\Theta)$ ,
2.  $h \in \text{Dom}(\Theta(s))$ , and
3.  $\Theta(s)|_h = \sigma \circ \{x \mapsto \alpha(z)\} \circ C|_h$  for some  $\sigma$ .

In an interpretation,  $rl(s)$  need not be  $r$ . The events that make up a strand's trace is all that matters. The protocol specific predicate  $P[r, h, x]$

is called a *strand progress predicate*, because it asserts a strand is associated with an instance of role  $r$  of height at least  $h$ .

The interpretation of the protocol independent predicates is straightforward.

- $k, \alpha \models \text{prec}(w, x, y, z)$  iff  $(\alpha(w), \alpha(x)) \prec (\alpha(y), \alpha(z))$ .
- $k, \alpha \models \text{non}(y)$  iff  $\alpha(y) \in N$ .
- $k, \alpha \models \text{uniq}(y)$  iff  $\alpha(y) \in U$ .
- $k, \alpha \models \text{orig}(x, y, z)$  iff  $\alpha(x) \in U$  and  $(\alpha(y), \alpha(z)) \in \mathcal{O}_k(\alpha(x))$ .
- $k, \alpha \models y = z$  iff  $\alpha(y) = \alpha(z)$ .
- $k, \alpha \not\models \text{false}$ .

**Theorem B.1.** Let  $\mathcal{F}(k_0) = (X, \Psi)$  and  $\Phi = \exists X \Psi$ . Formula  $\Phi$  is satisfied in  $k$  iff there is a homomorphism from  $k_0$  to  $k$ , i.e.  $k \models \Phi$  iff  $\exists \delta k_0 \xrightarrow{\delta} k$ .

This theorem corrects the first of the two main results from [6], as that paper omits the **orig** predicate.

*Proof.* For the forward direction, assume  $\alpha$  is a variable assignment for the variables in  $X$  such that  $k, \alpha \models \Psi$ , and let  $Z$  be the sequence of strand variables constructed while generating  $\Psi$  from  $k_0$ . Then the pair of maps  $\delta = (\alpha \circ Z, \alpha)$  demonstrate a homomorphism from  $k_0$  to  $k$ , i.e. each item in the definition of a preskeleton homomorphism on Page 12 is satisfied.

For the reverse direction, assume maps  $\delta = (\phi, \sigma)$  are such that  $k_0 \xrightarrow{\delta} k$ . Then the desired variable assignment is

$$\alpha(x) = \begin{cases} \phi(Z^{-1}(x)) & x \in \text{Ran}(Z) \\ \sigma(x) & x \in \text{Dom}(\sigma). \end{cases}$$

□

The set of bundles denoted by preskeleton  $k$ ,  $\llbracket k \rrbracket$  is defined on Page 13.

**Theorem B.2.** Let  $k_0 \xrightarrow{\delta_i} k_i$  be a complete set of homomorphisms for shapes  $k_i \in K$ , and assume  $\llbracket k_0 \rrbracket = \bigcup_{k \in K} \llbracket k \rrbracket$ . Then the shape analysis sentence  $\Sigma = \mathcal{S}(k_0 \xrightarrow{\delta_i} k_i)$  is satisfied in all realized skeletons  $k$ , i.e.  $k \models \Sigma$ .



*Proof.* Shapes are maximal among realized skeletons, so there is no realized skeleton in the image of  $k$  that is not in the image of one of the shapes. Therefore, by Theorem B.1, the negation of the hypothesis of the implication is satisfied in all realized skeletons that are not in the image of  $k_0$ , and the disjunction is satisfied in the remaining realized skeletons.  $\square$

The security goals of a protocol can be formalized using the same language used to specify shape analysis sentences. A security goal can express an authentication goal or a secrecy goal.

Security goals and shape analysis sentences can be translated into the language of ordinary first-order logic and used with an automated first-order theorem prover. If a theorem prover deduces security goal  $\Phi$  from shape analysis sentence  $\Sigma$ , then  $\Phi$  is satisfied in all realized skeletons.

Security goals can be used to ensure essential properties of a protocol are preserved in the face of changes to the protocol. Suppose an initial version of a protocol is specified, and shape analysis sentences for it are produced. The sentences can be edited to produce a formalization of security goals that should be preserved during any revision to the protocol. After a revision, one can generate revised shape analysis sentences, and use them to make sure each security goal is still deducible.

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